Decision Times in the Luce Model: An Experiment

Yukihiko Funaki Ruohan Ma Hendrik Rommeswinkel

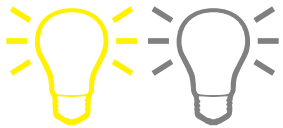
Waseda University

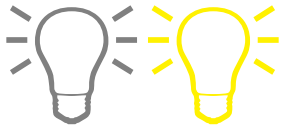
Oxford University

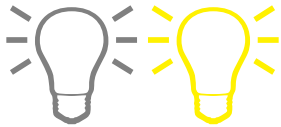
Hitotsubashi University and Waseda Institute for Advanced Study

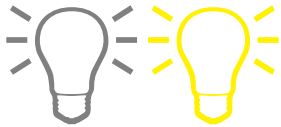
June 2024

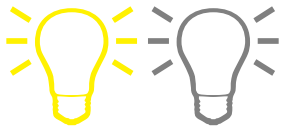
Table of Contents

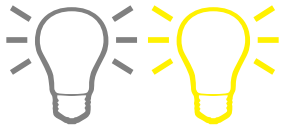


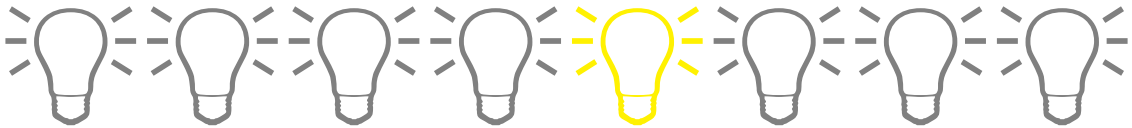


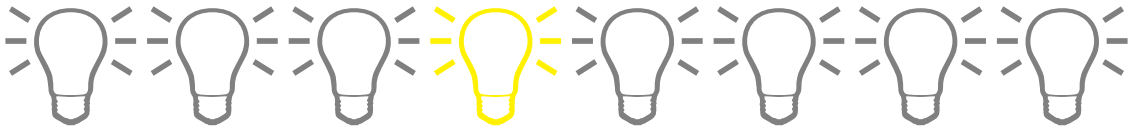


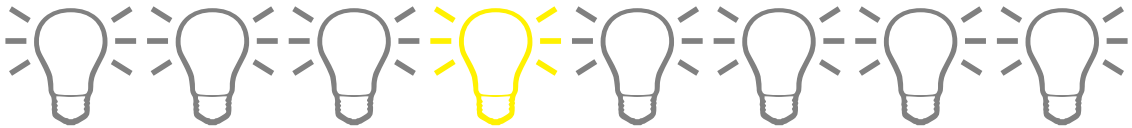






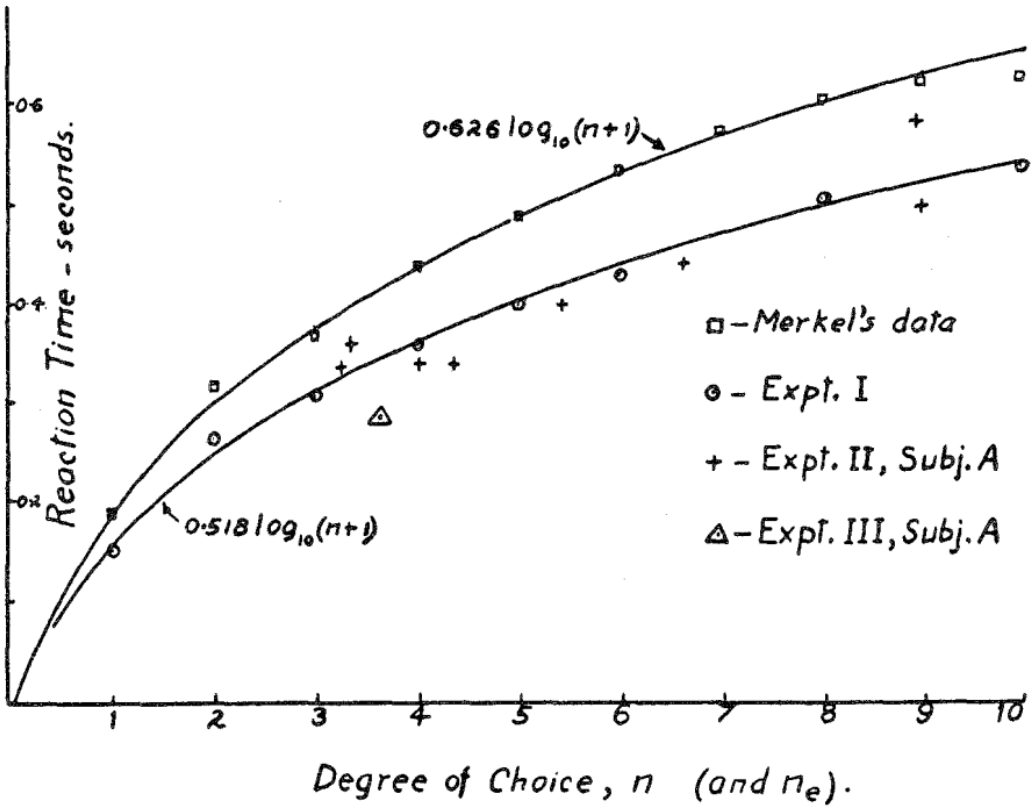






Hick's Law

Hick (1952) discovered a logarithmic relation between the number of options and the reaction time.



Shannon and Tsallis entropy

Entropy (Shannon, 1948; Tsallis, 1988)

$$h_r(p) = \begin{cases} -\sum_x p(x) \ln p(x), & r = 1 \\ (1 - \sum_x p(x)^r)/(r-1), & r \neq 1. \end{cases}$$

Extension of Hick's Law

- Is Hick's law purely a matter of *reaction*?
- Does Hick's law extend to *decisions*?
- Can we extend its information theoretic motivation to value tasks?


TANITA


T-fal


63.4
TANITA


T-fal


T-fal


63.4
kg
TANITA


63.4
kg
TANITA


COSORI


T-fal


ANKER

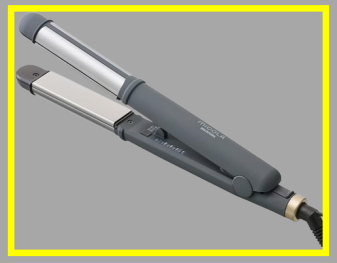

ANKER


soundcore


soundcore


T-fal


63.4
kg
TANITA




T-fal


soundcore


soundcore

Notation

- $\mathcal{X}$ denotes a universe of alternatives.
- $\mathcal{C}$ set of finite subsets of $\mathcal{X}$.
- Stochastic choice function $p : \mathcal{X} \times \mathcal{C} \to [0, 1]$ s.t. for all $C \in \mathcal{C}$: $x \notin C$, $p(x, C) = 0$ and $\sum_{x \in C} p(x, C) = 1$.

Decision Times

- $\tau : \mathcal{C} \to \mathbb{R}_+$ is a decision time.
- τ represents decision time relation $\succsim$ on $\mathcal{C}$:
- $\tau(C) \geq \tau(D) \Leftrightarrow C \succsim D$

Richness of Outcomes

Richness of Outcomes

The set of alternatives $\mathcal{X}$ fulfills richness if for every $x \in \mathcal{X}$ and every $\mu \in [0, 1]$ there is a countable number of alternatives $\{y_1, y_2, \ldots\}$ such that $p(x, \{x, y_i\}) = \mu$.

Continuity of Decision Times

Continuity of Decision Times

A stochastic choice function p and decision time τ fulfill continuity of decision times if for all sequences of opportunity sets $(A^k \equiv \{a_1^k, \ldots, a_n^k\})_{k=0}^{\infty}$ and $A = \{a_1, \ldots, a_m\}$, if $p(a_i^k, A^k) \to p(a_i, A)$ for all $i \in \{1, \ldots, m\}$ and $p(a_i^k, A^k) \to 0$ for all $i \in \{m+1, \ldots, n\}$ then $\tau(A^k) \to \tau(A)$.

Positivity

Positivity

A decision time τ fulfills positivity if for all $x, y \in \mathfrak{X}$, $\tau(\{x, y\}) > \tau(\{x\})$.

Independence of Decision Times

Independence of Decision Times

A stochastic choice function p and decision time τ fulfill independence of decision times if for all $C, D, E \in \mathcal{C}$ such that $(C \cup D) \cap E = \emptyset$ and $p(C, C \cup E) = p(D, D \cup E)$ it holds that

$$\begin{aligned} & \tau(C) \geq \tau(D) \\ \Leftrightarrow \quad & \tau(C \cup E) \geq \tau(D \cup E). \end{aligned} \tag{1}$$

Luce Model of Stochastic Choice

Positivity

For all $x, y \in \mathcal{X}$, $p(x, \{x, y\}) > 0$.

Independence of Irrelevant Alternatives (IIA)

For all $C \in \mathcal{C}$ and all $x, y \in C$:

$$p(x, \{x, y\}) / p(y, \{x, y\}) = p(x, C) / p(y, C)$$

Luce (1959)

Positivity and IIA imply existence of $v : \mathcal{X} \to \mathbb{R}$ s.t.

$$p(x, C) = \frac{\exp(v(x))}{\sum_{y \in C} \exp(v(y))}$$

Luce-Hick Model

Luce-Hick Model

A stochastic choice function $p : \mathcal{X} \times \mathcal{C} \to [0, 1]$ and a decision time $\tau : \mathcal{C} \to \mathbb{R}_+$ form a Luce-Hick model if

1. p follows the Luce model
2. $T(\tau(C)) = H_r(p(\cdot, C))$ where T is continuous and strictly monotone.

Luce-Hick Model

Luce-Hick Model

A stochastic choice function p and a decision time τ form a Luce-Hick model if

1. there exists a function $v : \mathcal{X} \to \mathbb{R}$ such that for all $C \in \mathcal{C}$ and $x \in C$,

$$p(x, C) = \frac{\exp(v(x))}{\sum_{y \in C} \exp(v(y))}, \text{ and}$$

2. there exists a continuous, strictly monotone function T and $r \in \mathbb{R}_{++}$ such that for all $C \in \mathcal{C}$,

$$T \circ \tau(C) = \begin{cases} \frac{1}{r-1}\left(1 - \sum_{x \in C} p(x, C)^r\right) & r \neq 1 \\ \sum_{x \in C} p(x, C) \ln p(x, C) & r = 1 \end{cases}$$

and $\tau(\{x\}) = \tau(\{y\}) = T^{-1}(0)$ for all $x, y \in \mathcal{X}$.

Luce-Hick Representation

Corollary

Suppose $\mathcal{X}$ fulfills richness in outcomes. Then the following statements are equivalent.

1. *p and τ fulfill Positivity, IIA, Independence of Decision Times, and Continuity of Decision Times.*
2. *p and τ form a Luce-Hick model.*

Betweenness

A relation representing decision times does not fulfill betweenness:

- $a \succsim b$...
- does not imply $a \succsim 1/2a \oplus 1/2b \succsim b$.

...but betweenness is implied by Independence and Reduction of Compound Lotteries.

Notation

- Sets in uppercase script, $\mathcal{S}$
- Elements of $\mathcal{S}$ in lowercase $a, b, c, \ldots$
- Real numbers $r > 0, q$.
- Real numbers λ, μ on the interval $[0, 1]$.
- $0 \ln 0 = 0$.

Procedural Mixture Set

$\mathcal{S}$ is a procedural mixture set if it fulfills for all $a, b, c \in \mathcal{S}$ the following conditions:

$$\begin{aligned}
\text{Closure:}\quad & \mu a \oplus (1-\mu) b \in \mathcal{S} \\
\text{Connectedness:}\quad & 1a \oplus (1-1) b = a, \\
\text{Commutativity:}\quad & \mu a \oplus (1-\mu) b = (1-\mu) b \oplus \mu a, \\
\text{Associativity:}\quad & (1-\lambda)\left[\frac{\mu}{1-\lambda} a \oplus \frac{1-\mu-\lambda}{1-\lambda} b\right] \oplus \lambda c \\
& = \mu a \oplus (1-\mu)\left[\frac{1-\mu-\lambda}{1-\mu} b \oplus \frac{\lambda}{1-\mu} c\right]
\end{aligned}$$

Expected Utility Axioms

Axiom 1: Weak Order

$\succsim$ is a complete, transitive relation on $\mathcal{S}$.

Axiom 2: Continuity

For any $a, b, c \in \mathcal{S}$, the sets $\{\mu | \mu a \oplus (1-\mu) b \succsim c\}$ and $\{\mu | c \succsim \mu a \oplus (1-\mu) b\}$ are closed.

Axiom 3: Independence

If $a, a', b \in \mathcal{S}$, $\mu \in (0,1)$ then $a \succsim a' \Leftrightarrow \mu a \oplus (1-\mu) b \succsim \mu a' \oplus (1-\mu) b$.

Theorem 1 (Rommeswinkel, *Procedural Mixture Sets*)

The relation $\succsim$ on the procedural mixture set $\mathcal{S}$ fulfills Axioms 1-3 if and only if there exists a continuous, real valued representation $U : \mathcal{S} \to \mathbb{R}$ such that,

$$U(\mu a \oplus (1-\mu)b) = \mu^r U(a) + (1-\mu)^r U(b) + q \cdot H_r(\mu, 1-\mu)$$

Table of Contents

Key features

- Relative attractiveness of options needs to be manipulable
- $|C| = 4$
- Time pressure
- Many incentivized choices

Decreasing Likelihood of Reception

- Each alternative x starts out with a certain probability $q(x)$ of actually receiving the good/payoff/etc. in case it is chosen.
- This probability decreases over time for every alternative until decision is made.
- Visualized via a central bar that becomes smaller. All items affected the same way.
- After 7 seconds, the central bar is empty and all alternatives have zero probability.

Choice Domain

1. Amazon electronic gadgets
2. Value between EUR 30 and EUR 50 (JPY 4800 - 8000)
3. Gadgets have color options
4. Each option is a binary lottery between receiving the good or receiving nothing.
5. Winning probabilities 0.4, 0.5, 0.6, 0.7, 0.8, or 0.9

Procedures

- Experiments conducted at U Konstanz (1+2) and Waseda U (3+4)
- Recruitment via ORSEE (Konstanz)
- Experiment software oTree
- Show up fee of 3 EUR in Konstanz
- Completion fee of 10 EUR in Konstanz, 2000 JPY at Waseda
- Approximately 1/2 chance of winning an item.

Sessions

- Pilot 1: 5 subjects, 1300 decisions.
- Gadget colors: black x 2, white x 2, grey, red, blue
- Pilot 2: 5 subjects, 1400 decisions.
- Gadget colors: black x 2, white x 2, red x 2, grey
- Pilot 3: 10 subjects, 1200 decisions.
- Gadget colors: black x 2, white x 2, red x 2, grey black x 5, white x 3, red x 2, green x 3, beige x 4, pink x 2, blue x 6
- Pilot 4: 10 subjects, 1200 decisions.
- Gadget colors: black x 2, beige x 2, blue x 2, grey, red
- Subjects finished after 1:30 to 2:00 hours with one exception.

Table of Contents

Empirical Methodology

- Estimation of choice probabilities from Luce model
- Choose "best" Tsallis index
- Separate permutation test for every subject whether entropy of decision probabilities is rank correlated to decision time.

Functional Form of Luce Utilities

The utility of a lottery depends both on the item x and the probability of receiving the item, q. q depends in turn on both the item x and the time at which the decision is made, t.

$$v(q, x) = u(x) + \beta \ln q$$

is equivalent to time-invariance of decision probability.

Luce Model Estimation

$$\ln p(x, C) = \beta \ln(q(x)) + u(x) - \ln(\sum_{x \in C} q(x)^{\beta} \cdot \exp(u(x)))$$

where:

- $p(x, C)$: stochastic choice function
- $q(x)$: probability of receiving x, if chosen.
- β: elasticity wrt. this probability.
- $u(x)$: subjective utility of receiving x.

The Challenge

"Zuerst habe ich [mich] auf Kocher und Heißluftfriteuse fokussiert. Dann ist das "langweilig" [geworden]. Dann habe ich [mich] auf die höchsten blauen Balken konzentriert und als das wieder langweilig wurde hab ich einfach nur gedrückt." - Subject 1myidkd3

Luce Model Estimation - P1

Table: MLE Estimation Results Session 1

session	participant	toothbrush black	speaker blue	powerbank white	mouse red	kettle grey	airpurifier white	airfryer black	beta	lln	rmsq
1	1myidkd3	0.09 [-0.22, 0.43]	-0.37 [-0.71, -0.02]	-0.76 [-1.09, -0.44]	-0.81 [-1.18, -0.46]	0.62 [0.32, 0.93]	-0.82 [-1.18, -0.51]	1.27 [1.00, 1.53]	1	1455.29	-0.64
	9bdtdh7l	-4.95 [-5.74, -4.29]	-5.51 [-6.27, -4.89]	-3.04 [-3.66, -2.57]	-7.22 [-18.77, -6.41]	-6.20 [-7.35, -5.39]	-0.32 [-0.76, 0.18]	0.87 [0.43, 1.34]	1	474.84	-0.32
	l28kzk6x	-2.31 [-2.80, -1.86]	-2.76 [-3.39, -2.21]	1.60 [1.24, 2.03]	-4.75 [-6.20, -3.87]	-2.69 [-3.30, -2.15]	-0.55 [-0.98, -0.13]	0.65 [0.35, 1.00]	1	776.58	-0.43
	lqvgn3ve	-1.96 [-2.47, -1.51]	-2.76 [-3.33, -2.20]	-3.66 [-4.33, -3.09]	-5.99 [-18.31, -4.99]	-4.92 [-6.11, -4.20]	1.29 [0.86, 1.74]	1.72 [1.30, 2.18]	1	544.63	-0.35
	ogo619di	-9.26 [-20.57, -7.70]	-3.44 [-5.07, -2.67]	-3.60 [-5.26, -2.84]	-4.21 [-5.89, -3.50]	-7.94 [-15.86, -6.96]	-6.76 [-8.58, -5.89]	1.97 [1.57, 2.45]	1	415.63	-0.32

Luce Model Estimation - P2

Table: MLE Estimation Results Session 2

session	participant	speaker red	powerbank white	powerbank black	mouse red	kettle grey	airpurifier white	airfryer black	beta	lln	rmsq
2	dluan0r0	5.71 [5.18, 6.33]	1.51 [0.84, 2.08]	1.37 [0.63, 1.90]	7.02 [6.43, 7.77]	2.21 [1.63, 2.77]	3.35 [2.84, 3.86]	5.05 [4.58, 5.58]	1	805.84	-0.43
	lq98fyew	2.84 [2.36, 3.31]	-3.57 [-15.81, -2.46]	-0.73 [-1.25, -0.25]	2.45 [2.04, 2.93]	1.10 [0.70, 1.49]	-4.38 [-15.99, -3.16]	3.93 [3.51, 4.41]	1	778.61	-0.42
	w3jkdjz8	5.70 [5.17, 6.35]	6.62 [6.06, 7.31]	6.11 [5.58, 6.78]	4.08 [3.54, 4.73]	3.11 [2.51, 3.78]	3.41 [2.86, 4.04]	2.79 [2.47, 3.15]	1	1097.59	-0.51
	w5zcovkt	-2.39 [-2.81, -1.97]	-2.42 [-2.87, -2.01]	-1.87 [-2.26, -1.55]	-2.27 [-2.67, -1.84]	0.19 [-0.11, 0.51]	-1.55 [-1.90, -1.21]	1.08 [0.80, 1.39]	1	1124.72	-0.51
	yhf9pvbv	4.67 [4.21, 5.14]	1.79 [1.32, 2.30]	1.89 [1.43, 2.32]	3.15 [2.70, 3.60]	3.90 [3.47, 4.32]	4.22 [3.81, 4.62]	5.58 [5.16, 6.03]	1	1062.38	-0.51

Luce Model Estimation - P3

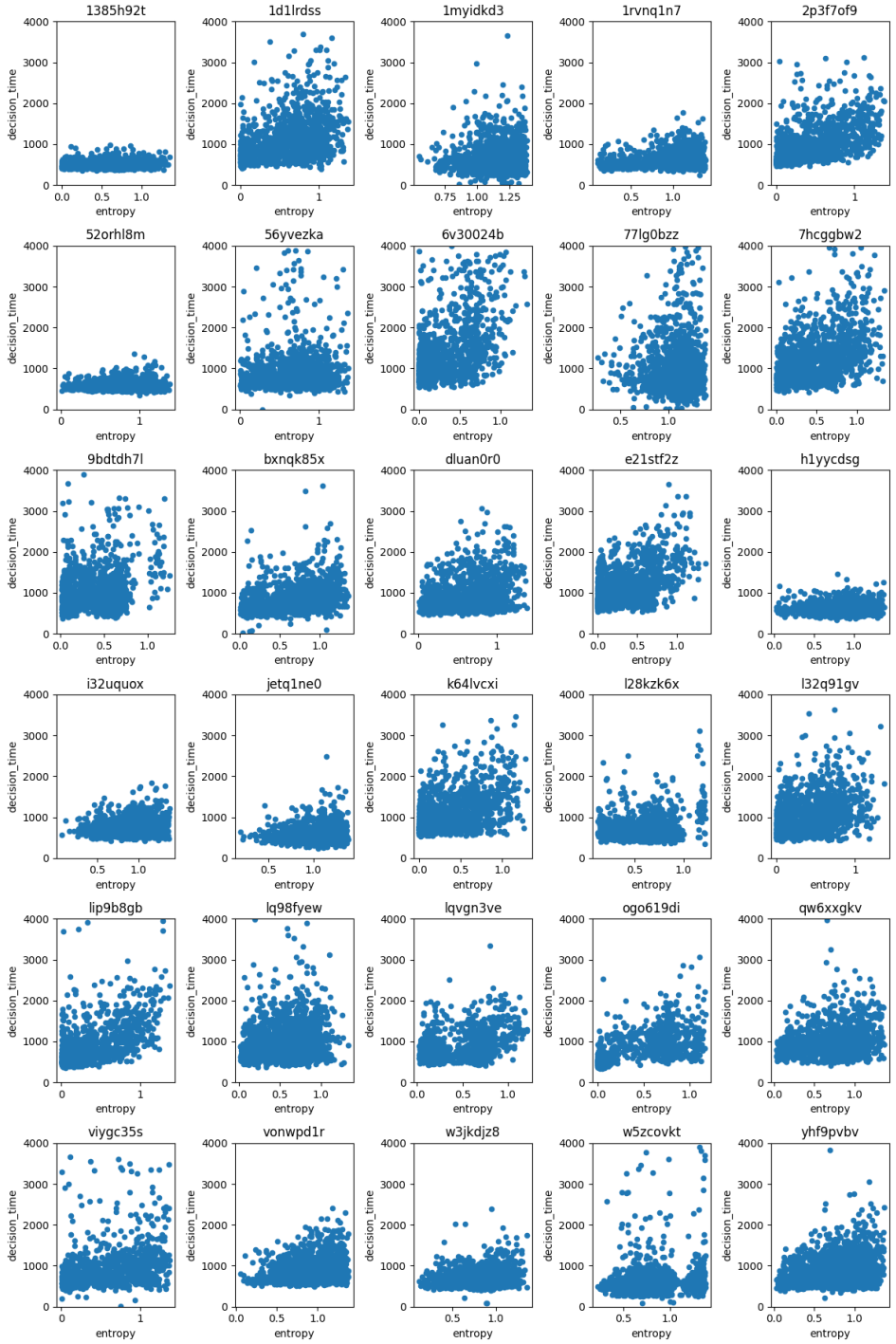
Table: MLE Estimation Results Session 3

session	participant	toothbrush pink	toothbrush blue	toaster red	toaster beige	straightiron blue	straightiron beige	speaker red	speaker blue	scale white	scale black	powerbank green	powerbank blue	keyboard white	keyboard black	kettle white	kettle green	kettle blue	kettle black	earphones black	earphones beige	blender pink	blender green	blender blue	blender beige	airfryer white	airfryer black	beta	lln	rmsq
3	1385h92t	4.88	4.96	6.07	5.84	5.17	5.04	5.21	5.41	5.28	5.25	5.21	5.69	5.51	5.63	5.66	5.50	5.22	6.00	5.58	5.66	5.35	4.86	5.57	5.20	5.68	5.57	1	744.62	-0.45
		[4.12, 5.92]	[4.19, 5.93]	[5.28, 7.25]	[5.02, 7.06]	[4.41, 6.20]	[4.29, 6.02]	[4.46, 6.23]	[4.69, 6.47]	[4.60, 6.31]	[4.44, 6.34]	[4.52, 6.27]	[4.92, 6.81]	[4.77, 6.70]	[4.83, 6.84]	[4.87, 6.74]	[4.75, 6.57]	[4.44, 6.44]	[5.19, 7.07]	[4.83, 6.69]	[4.90, 6.77]	[4.67, 6.46]	[4.14, 6.00]	[4.78, 6.69]	[4.55, 6.28]	[4.94, 6.73]	[5.00, 6.46]			
	1d1lrdss	2.17	3.18	3.18	3.22	10.33	9.57	5.15	5.64	3.86	3.83	4.46	5.46	5.04	4.37	5.75	3.52	3.88	5.64	3.15	2.81	2.22	3.62	3.77	4.29	2.59	2.79	1	732.81	-0.44
		[1.13, 3.12]	[2.41, 4.08]	[2.30, 4.08]	[2.38, 4.12]	[9.19, 12.08]	[8.52, 10.97]	[4.44, 6.07]	[4.96, 6.63]	[3.10, 4.75]	[3.16, 4.71]	[3.69, 5.38]	[4.82, 6.39]	[4.28, 6.01]	[3.65, 5.29]	[5.06, 6.73]	[2.79, 4.46]	[3.11, 4.84]	[4.94, 6.59]	[2.50, 4.13]	[1.97, 3.78]	[1.34, 3.10]	[2.99, 4.56]	[3.14, 4.71]	[3.56, 5.20]	[1.71, 3.59]	[2.50, 3.24]			
	1rvnq1n7	1.99	2.29	1.87	1.29	2.57	2.78	2.98	2.13	1.41	1.36	5.10	4.98	1.20	1.05	1.59	1.72	1.97	1.47	1.72	2.43	1.62	1.72	1.95	1.30	0.94	1.35	1	1154.85	-0.59
		[1.47, 2.59]	[1.72, 2.95]	[1.31, 2.56]	[0.69, 2.01]	[2.05, 3.26]	[2.30, 3.43]	[2.44, 3.66]	[1.57, 2.82]	[0.69, 2.09]	[0.78, 1.97]	[4.51, 5.95]	[4.40, 5.85]	[0.60, 1.84]	[0.34, 1.71]	[1.05, 2.22]	[1.24, 2.32]	[1.44, 2.58]	[0.95, 2.18]	[1.12, 2.36]	[1.91, 2.99]	[1.01, 2.31]	[1.22, 2.41]	[1.39, 2.61]	[0.71, 1.94]	[0.20, 1.62]	[1.20, 1.52]			
	2p3f7of9	7.40	7.30	2.15	1.44	6.14	5.44	10.71	11.72	1.67	0.93	2.77	3.15	2.13	2.77	2.25	5.15	5.52	3.98	9.41	7.31	2.00	2.15	2.38	2.40	0.63	1.03	1	516.56	-0.36
		[6.46, 13.33]	[6.36, 13.64]	[1.10, 7.61]	[0.02, 7.86]	[5.26, 12.36]	[4.59, 11.58]	[9.68, 17.23]	[10.60, 18.07]	[0.48, 7.84]	[-7.55, 6.76]	[1.76, 9.23]	[2.14, 9.32]	[0.71, 8.17]	[1.85, 8.96]	[1.06, 8.69]	[4.26, 11.12]	[4.68, 11.81]	[3.14, 10.12]	[8.39, 15.47]	[6.28, 13.46]	[0.87, 7.96]	[1.13, 8.54]	[1.39, 8.44]	[1.32, 8.63]	[-7.01, 6.51]	[0.84, 1.26]			
	56yvezka	4.14	3.46	4.17	4.19	3.85	3.72	6.33	6.29	3.94	4.11	5.34	6.00	3.89	3.96	4.46	3.93	3.94	4.36	5.34	5.08	3.98	3.54	3.86	4.09	4.03	4.37	1	790.22	-0.46
		[3.41, 4.97]	[2.76, 4.34]	[3.43, 5.00]	[3.44, 5.09]	[3.07, 4.70]	[2.97, 4.55]	[5.61, 7.30]	[5.56, 7.21]	[3.22, 4.77]	[3.47, 4.92]	[4.55, 6.28]	[5.20, 6.99]	[3.18, 4.65]	[3.21, 4.87]	[3.78, 5.22]	[3.20, 4.73]	[3.12, 4.79]	[3.62, 5.20]	[4.56, 6.24]	[4.29, 5.93]	[3.17, 4.88]	[2.80, 4.35]	[3.19, 4.57]	[3.34, 4.97]	[3.21, 4.77]	[3.97, 4.97]			
	7hcggbw2	-0.47	-1.03	3.52	2.98	1.54	1.23	6.53	5.87	4.62	4.02	7.27	6.66	0.81	1.42	1.51	-1.13	-0.21	1.06	4.22	4.80	-2.67	-2.91	-2.10	-1.85	2.02	1.58	1	594.11	-0.39
		[-1.73, 0.54]	[-2.87, 0.10]	[2.93, 4.38]	[2.28, 3.95]	[0.84, 2.35]	[0.45, 2.17]	[5.89, 7.59]	[5.14, 6.88]	[3.92, 5.61]	[3.36, 4.91]	[6.55, 8.37]	[6.01, 7.80]	[0.02, 1.56]	[0.68, 2.26]	[0.83, 2.33]	[-2.87, -0.12]	[-1.40, 0.76]	[0.17, 1.92]	[3.54, 5.11]	[4.12, 5.72]	[-15.51, -1.13]	[-15.41, -1.25]	[-16.03, -0.74]	[-11.28, -0.63]	[1.25, 2.90]	[1.39, 1.88]			
	i32uquox	-1.31	-1.09	0.77	0.46	-0.90	-1.07	-0.30	-0.11	-0.52	0.36	0.71	-0.03	-0.49	-0.39	0.87	1.34	1.03	1.78	0.34	-0.55	-1.45	-1.00	-1.24	-1.12	0.78	1.78	1	1125.30	-0.58
		[-1.98, -0.72]	[-1.70, -0.54]	[0.38, 1.25]	[-0.01, 0.97]	[-1.52, -0.31]	[-1.72, -0.49]	[-0.82, 0.24]	[-0.69, 0.49]	[-1.03, 0.02]	[-0.10, 0.90]	[0.23, 1.29]	[-0.56, 0.55]	[-1.14, 0.01]	[-0.95, 0.20]	[0.37, 1.47]	[0.89, 1.93]	[0.53, 1.62]	[1.29, 2.35]	[-0.17, 0.89]	[-1.09, 0.01]	[-2.20, -0.86]	[-1.58, -0.41]	[-2.00, -0.68]	[-1.67, -0.53]	[0.34, 1.33]	[1.62, 2.00]			
	k64lvcxi	-0.48	-0.14	6.95	7.20	8.80	8.83	7.75	7.44	3.98	4.37	7.39	7.57	10.13	9.56	2.85	2.36	2.23	2.86	8.81	9.14	1.34	1.10	2.48	2.19	1.67	3.01	1	518.79	-0.36
		[-11.12, 1.13]	[-12.63, 1.67]	[6.01, 8.57]	[6.20, 8.91]	[7.72, 10.69]	[7.83, 10.66]	[6.70, 9.41]	[6.42, 9.19]	[2.87, 5.50]	[3.38, 5.87]	[6.37, 9.08]	[6.60, 9.29]	[8.97, 12.12]	[8.54, 11.46]	[1.75, 4.16]	[1.05, 3.82]	[1.11, 3.49]	[1.58, 4.33]	[7.75, 10.61]	[8.07, 11.04]	[-0.41, 2.74]	[-0.50, 2.60]	[1.12, 3.95]	[0.82, 3.67]	[0.01, 3.50]	[2.74, 3.48]			
	qw6sxgkv	1.28	1.65	4.50	4.29	0.19	-0.34	1.34	0.95	1.53	1.61	0.97	1.20	-0.97	-0.62	2.77	0.30	0.91	2.61	-0.30	-0.90	1.65	1.77	1.50	1.80	2.67	2.70	1	868.68	-0.49
		[0.70, 1.86]	[1.12, 2.28]	[3.97, 5.25]	[3.66, 5.03]	[-0.53, 0.96]	[-1.05, 0.35]	[0.76, 1.96]	[0.27, 1.58]	[0.95, 2.22]	[0.98, 2.33]	[0.36, 1.65]	[0.61, 1.78]	[-2.18, -0.13]	[-1.70, 0.16]	[2.29, 3.39]	[-0.35, 0.98]	[0.24, 1.49]	[2.10, 3.21]	[-1.08, 0.44]	[-1.80, -0.20]	[1.10, 2.32]	[1.25, 2.34]	[0.95, 2.13]	[1.30, 2.38]	[2.17, 3.34]	[2.46, 3.04]			
	vonwpd1r	-0.12	-0.13	0.44	0.98	2.34	2.37	-0.25	0.23	-0.46	-0.20	0.50	0.41	1.57	1.31	0.89	0.72	0.64	0.73	2.68	3.43	2.81	2.62	2.55	2.13	1.79	1.62	1	1070.59	-0.56
		[-0.91, 0.56]	[-0.84, 0.51]	[-0.26, 1.07]	[0.48, 1.56]	[1.81, 2.91]	[1.89, 2.89]	[-0.98, 0.39]	[-0.38, 0.86]	[-1.30, 0.24]	[-1.02, 0.48]	[-0.12, 1.08]	[-0.25, 0.98]	[1.04, 2.13]	[0.78, 1.84]	[0.33, 1.47]	[0.05, 1.29]	[-0.00, 1.21]	[0.09, 1.30]	[2.14, 3.35]	[2.95, 4.08]	[2.29, 3.39]	[2.11, 3.25]	[2.09, 3.17]	[1.63, 2.73]	[1.31, 2.38]	[1.46, 1.84]			

Luce Model Estimation - P4

Table: MLE Estimation Results Session 4

session	participant	straightiron beige	speaker red	speaker grey	powerbank blue	kettle blue	kettle black	blender beige	airfryer black	beta	lln	rmsq
4	52orhl8m	3.41 [2.93, 4.03]	3.25 [2.74, 3.87]	3.08 [2.57, 3.66]	3.74 [3.16, 4.38]	3.57 [3.08, 4.17]	4.03 [3.52, 4.68]	3.42 [2.91, 3.99]	3.60 [3.20, 4.10]	1	757.63	-0.53
	6v30024b	-1.76 [-3.37, -0.92]	8.59 [7.59, 9.95]	8.54 [7.60, 9.86]	0.25 [-0.71, 1.07]	2.64 [2.02, 3.30]	6.73 [5.96, 7.80]	1.20 [0.35, 1.91]	4.38 [3.87, 5.08]	1	396.94	-0.31
	77lg0bzz	1.53 [1.22, 1.92]	2.68 [2.33, 3.04]	1.52 [1.19, 1.86]	1.89 [1.57, 2.24]	1.45 [1.10, 1.82]	1.76 [1.44, 2.10]	1.29 [0.96, 1.68]	1.73 [1.52, 1.96]	1	1285.37	-0.61
	bxnqk85x	2.71 [2.21, 3.25]	6.96 [6.30, 7.80]	6.54 [5.94, 7.30]	3.83 [3.32, 4.42]	3.22 [2.72, 3.77]	3.58 [3.13, 4.11]	2.82 [2.29, 3.39]	3.14 [2.82, 3.60]	1	727.47	-0.44
	e21stf2z	7.23 [6.32, 8.36]	11.77 [10.43, 13.48]	11.37 [10.12, 13.05]	12.50 [11.15, 14.23]	5.29 [4.47, 6.24]	5.86 [5.08, 6.91]	4.53 [3.44, 5.53]	4.76 [4.19, 5.57]	1	404.68	-0.32
	h1yycdsg	3.07 [2.69, 3.53]	4.62 [4.17, 5.20]	3.68 [3.27, 4.21]	3.34 [2.92, 3.81]	3.27 [2.83, 3.79]	3.62 [3.19, 4.12]	3.04 [2.61, 3.53]	3.11 [2.83, 3.46]	1	1019.43	-0.54
	jetq1ne0	2.22 [1.91, 2.59]	1.81 [1.49, 2.20]	1.94 [1.62, 2.30]	2.44 [2.13, 2.86]	2.38 [2.07, 2.77]	2.41 [2.12, 2.80]	3.41 [3.08, 3.82]	1.86 [1.68, 2.06]	1	1234.23	-0.61
	l32q91gv	-1.34 [-2.10, -0.75]	2.43 [2.01, 2.97]	3.13 [2.70, 3.72]	-0.68 [-1.32, -0.08]	0.31 [-0.16, 0.83]	-0.21 [-0.76, 0.29]	3.51 [3.03, 4.13]	5.36 [4.89, 5.99]	1	499.13	-0.36
	lip9b8gb	-4.22 [-5.13, -3.63]	-1.86 [-2.24, -1.50]	-1.43 [-1.75, -1.08]	-2.77 [-3.22, -2.32]	-4.25 [-5.22, -3.58]	-4.07 [-4.82, -3.51]	4.08 [3.49, 5.05]	0.92 [0.71, 1.21]	1	421.95	-0.30
	viygc35s	1.88 [1.37, 2.42]	7.39 [6.58, 8.39]	6.71 [5.96, 7.68]	2.54 [2.08, 3.12]	1.93 [1.36, 2.50]	2.50 [2.03, 3.06]	1.56 [0.96, 2.11]	2.34 [2.06, 2.72]	1	596.55	-0.39


1385h92t
1d1lrdss
1myidkd3
1rvnq1n7
2p3f7of9
52orhl8m
56yvezka
6v30024b
77lg0bzz
7hcggbw2
9bdtdh7l
bxnqk85x
dluan0r0
e21stf2z
h1yycdsg
i32uquox
jetq1ne0
k64lvcxi
l28kzk6x
l32q91gv
lip9b8gb
lq98fyew
lqvgn3ve
ogo619di
qw6xxgkv
viygc35s
vonwpd1r
w3jkdjz8
w5zcovkt
yhf9pvbv
decision_time
entropy

Tsallis Entropy Index

		Tsallis' q
session	participant	
1	1myidkd3	37.04, (177.86), [0.00, 129.06]
	9bdtdh7l	0.00, (0.00), [0.00, 0.00]
	l28kzk6x	0.00, (0.00), [0.00, 0.02]
	lqvgn3ve	0.39, (0.07), [0.23, 0.50]
	ogo619di	138.20, (562.24), [0.00, 148.35]
2	dluan0r0	0.16, (0.31), [0.00, 0.37]
	lq98fyew	0.00, (0.00), [0.00, 0.00]
	w3jkdjz8	0.00, (0.00), [0.00, 0.07]
	w5zcovkt	0.73, (2.47), [0.00, 39.58]
	yhf9pvbv	0.00, (0.00), [0.00, 0.05]
3	1385h92t	0.41, (0.44), [0.05, 2.09]
	1d1lrdss	136.73, (404.83), [0.14, 148.33]
	1rvnq1n7	142.26, (18.40), [95.34, 147.30]
	2p3f7of9	0.00, (0.00), [0.00, 0.02]
	56yvezka	0.05, (0.21), [0.00, 136.68]
	7hcggbw2	0.00, (0.00), [0.00, 0.08]
	i32uquox	0.00, (0.00), [0.00, 107.36]
	k64lvcxi	0.00, (0.00), [0.00, 0.02]
	qw6xxgkv	0.15, (0.43), [0.00, 0.37]
	vonwpd1r	0.07, (0.16), [0.00, 145.16]
4	52orhl8m	0.08, (0.33), [0.00, 139.05]
	6v30024b	0.09, (0.07), [0.01, 0.18]
	77lg0bzz	65.09, (102.47), [0.19, 139.39]
	bxnqk85x	0.59, (0.53), [0.28, 51.86]
	e21stf2z	0.18, (0.04), [0.10, 0.23]
	h1yycdsg	0.00, (0.00), [0.00, 4.59]
	jetq1ne0	69.34, (349.24), [0.00, 107.13]
	l32q91gv	0.19, (0.06), [0.10, 0.33]
	lip9b8gb	0.00, (0.01), [0.00, 0.11]
	viygc35s	0.58, (0.62), [0.25, 73.87]

Permutation Test Methodology

- Permutation test:
- Test Statistic: Spearman's rho
- p-values:
  - Randomly permute decision times
  - Choose entropy index to maximize rank correlation
  - Calculate rank correlation
  - Repeat 10 000 times to obtain distribution of test statistic under null hypothesis

Spearman Rank Correlation Entropy - Decision Time

session	participant	Shannon model	Shannon p-value	Tsallis model	Tsallis p-value
1	1myidkd3	0.07	0.01	0.08	0.000
	9bdtdh7l	0.17	0.00	0.34	0.000
	l28kzk6x	0.07	0.01	0.14	0.000
	lqvgn3ve	0.38	0.00	0.40	0.000
	ogo619di	0.69	0.00	0.70	0.000
2	dluan0r0	0.37	0.00	0.39	0.000
	lq98fyew	0.18	0.00	0.24	0.000
	w3jkdjz8	0.20	0.00	0.24	0.000
	w5zcovkt	0.20	0.00	0.20	0.000
	yhf9pvbv	0.27	0.00	0.32	0.000
3	1385h92t	0.17	0.00	0.17	0.000
	1d1lrdss	0.43	0.00	0.43	0.000
	1rvnq1n7	0.19	0.00	0.25	0.000
	2p3f7of9	0.65	0.00	0.72	0.000
	56yvezka	0.13	0.00	0.15	0.000
	7hcggbw2	0.45	0.00	0.52	0.000
	i32uquox	0.10	0.00	0.10	0.000
	k64lvcxi	0.36	0.00	0.50	0.000
	qw6xxgkv	0.27	0.00	0.29	0.000
	vonwpd1r	0.26	0.00	0.27	0.000
4	52orhl8m	0.08	0.02	0.08	0.000
	6v30024b	0.48	0.00	0.56	0.000
	77lg0bzz	0.05	0.08	0.08	0.000
	bxnqk85x	0.47	0.00	0.47	0.000
	e21stf2z	0.51	0.00	0.57	0.000
	h1yycdsg	0.10	0.00	0.13	0.000
	jetq1ne0	0.05	0.08	0.08	0.000
	l32q91gv	0.40	0.00	0.44	0.000
	lip9b8gb	0.61	0.00	0.64	0.000
	viygc35s	0.45	0.00	0.46	0.000

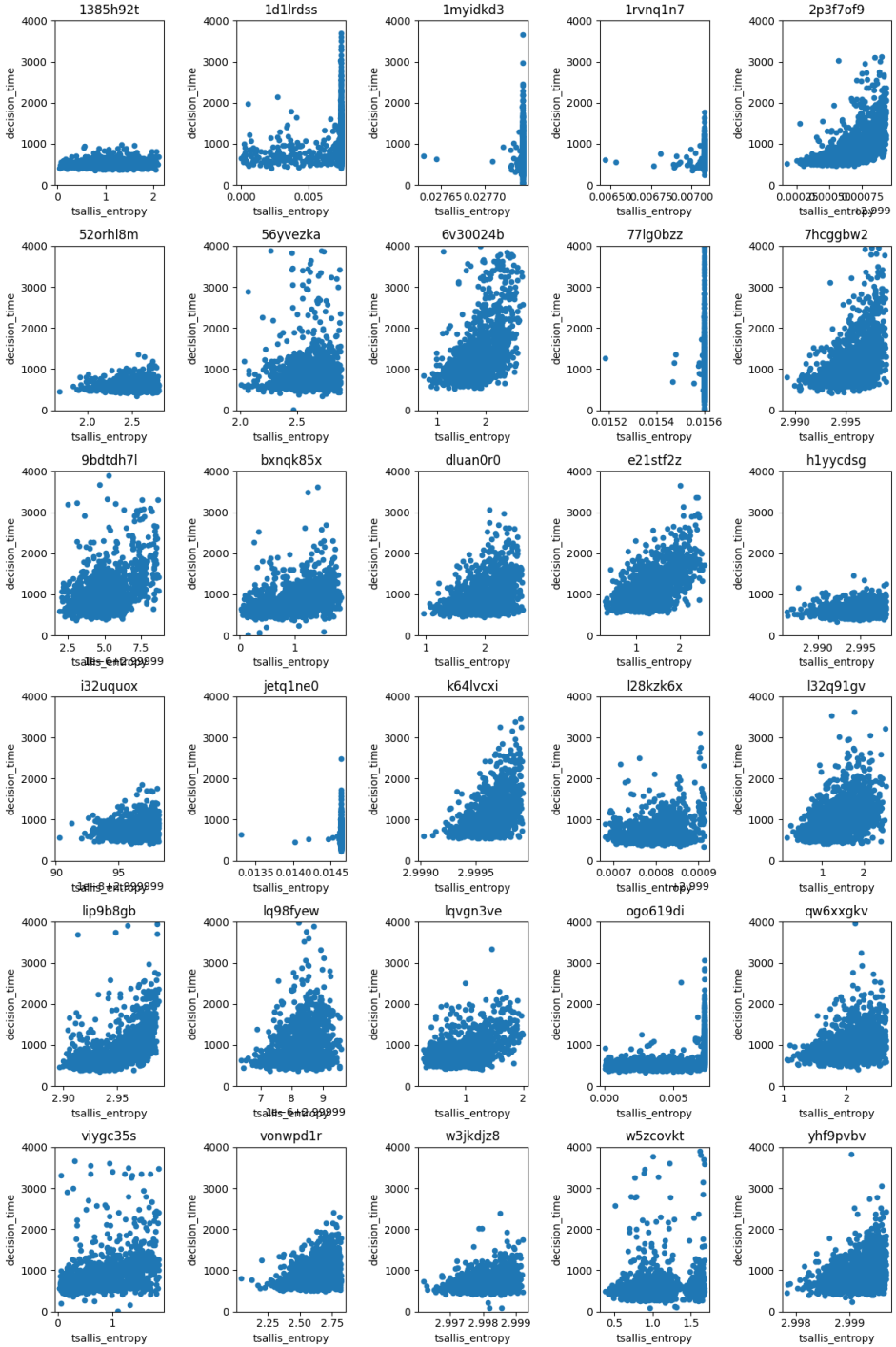

1385h92t
1d1lrdss
1myidkd3
1rvnq1n7
2p3f7of9
52orhl8m
56yvezka
6v30024b
77lg0bzz
7hcggbw2
9bdtdh7l
bxnqk85x
dluan0r0
e21stf2z
h1yycdsg
i32uquox
jetq1ne0
k64lvcxi
l28kzk6x
l32q91gv
lip9b8gb
lq98fyew
lqvgn3ve
ogo619di
qw6xxgkv
viygc35s
vonwpd1r
w3jkdjz8
w5zcovkt
yhf9pvbv
decision_time
tsallis_entropy

Table of Contents

Theoretical Literature

- Diffusion Drift Model: (Baldassi et al., 2020; Fudenberg et al., 2020; Ratcliff, 1978),
- Multialternative choice: (Cerreia-Vioglio et al., 2022; Krajbich & Rangel, 2011; McMillen & Holmes, 2005; Tajima et al., 2019).
- Binary Choice: (Echenique & Saito, 2017; Koida, 2017)
- "Behavioral" decision times: (Achtziger & Alós-Ferrer, 2014; Alós-Ferrer, 2018; Chabris et al., 2009; Drugowitsch et al., 2012; Fudenberg et al., 2018)

Empirical Literature

- Diffusion Drift Model: (Drugowitsch et al., 2012; Ratcliff, 1978; Ratcliff & McKoon, 2008),
- Consumption choices: (Ashby et al., 2016; Clithero, 2018; Krajbich & Rangel, 2011; Krajbich et al., 2012; Reutskaja et al., 2011)
- Electric household items: Krajbich et al. (2012)
- Hick's Law: (Brainard et al., 1962; Hick, 1952; Hyman, 1953; Kornblum, 1969)

Table of Contents

Conclusions

- Extension of Hick's law to entropy of endogeneous choice probabilities.

Conclusions

- Extension of Hick's law to entropy of endogeneous choice probabilities.
- Entropy of choice probabilities highly predictive of decision times

Conclusions

- Extension of Hick's law to entropy of endogeneous choice probabilities.
- Entropy of choice probabilities highly predictive of decision times
- Hick's law naturally extends to choice probabilities from the Luce model but entropy index somewhat arbitrary.

Conclusions

- Extension of Hick's law to entropy of endogeneous choice probabilities.
- Entropy of choice probabilities highly predictive of decision times
- Hick's law naturally extends to choice probabilities from the Luce model but entropy index somewhat arbitrary.
- Much less data than 1200 decisions would be needed.

Conclusions

- Extension of Hick's law to entropy of endogeneous choice probabilities.
- Entropy of choice probabilities highly predictive of decision times
- Hick's law naturally extends to choice probabilities from the Luce model but entropy index somewhat arbitrary.
- Much less data than 1200 decisions would be needed.
- What induces greater variance of decision time at higher entropy?

Thank You!

Achtziger, A., & Alós-Ferrer, C. (2014).Fast or rational? a response-times study of bayesian updating. *Management Science*, *60*(4), 923–938.

Alós-Ferrer, C. (2018).A dual-process diffusion model. *Journal of Behavioral Decision Making*, *31*, 203–218.

Ashby, N. J., Jekel, M., Dickert, S., & Glöckner, A. (2016).Finding the right fit: A comparison of process assumptions underlying popular drift-diffusion models.. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *42*(12), 1982.

Baldassi, C., Cerreia-Vioglio, S., Maccheroni, F., Marinacci, M., & Pirazzini, M. (2020).A behavioral characterization of the drift diffusion model and its multialternative extension for choice under time pressure. *Management Science*, *66*(11), 5075–5093.

Brainard, R. W., Irby, T. S., Fitts, P. M., & Alluisi, E. A. (1962).Some variables influencing the rate of gain of information.. *Journal of experimental psychology*, *63*(2), 105.

Cerreia-Vioglio, S., Maccheroni, F., Marinacci, M., & Rustichini, A. (2022).Multinomial logit processes and preference discovery: Inside

and outside the black box. *The Review of Economic Studies*, *90*(3), 1155–1194.

Chabris, C., Laibson, D., Morris, C. L., Schuldt, J. P., & Taubinsky, D. (2009).The allocation of time in decision-making. *Journal of the European Economic Association*, *7*(2), 628–637.

Clithero, J. A. (2018).Improving out-of-sample predictions using response times and a model of the decision process. *Journal of Economic Behavior & Organization*, *148*, 344–375.

Drugowitsch, J., Moreno-Bote, R., Churchland, A. K., Shadlen, M. N., & Pouget, A. (2012).The cost of accumulating evidence in perceptual decision making. *Journal of Neuroscience*, *32*(11), 3612–3628.

Echenique, F., & Saito, K. (2017).Response time and utility. *Journal of Economic Behavior and Organization*, *139*, 49–59.

Fudenberg, D., Newey, W., Strack, P., & Strzalecki, T. (2020).Testing the drift-diffusion model. *Proceedings of the National Academy of Sciences*, *117*(52), 33141–33148.

Fudenberg, D., Strack, P., & Strzalecki, T. (2018).Speed, accuracy, and the optimal timing of choices. *American Economic Review*, *108*(12), 3651–3684.

Hick, W. (1952).On the rate of gain of information. *Quarterly Journal of Experimental Psychology*, *4*(1), 11–26.

Hyman, R. (1953).Stimulus information as a determinant of reaction time.. *Journal of experimental psychology*, *45*(3), 188.

Koida, N. (2017).A multiattribute decision time theory. *Theory and Decision*, *83*, 407–430.

Kornblum, S. (1969).Sequential determinants of information processing in serial and discrete choice reaction time.. *Psychological Review*, *76*(2), 113.

Krajbich, I., & Rangel, A. (2011).Multialternative drift-diffusion model predicts the relationship between visual fixations and choice in value-based decisions. *PNAS*, *108*(33), 13852–13857.

Krajbich, I., Lu, D., Camerer, C., & Rangel, A. (2012).The attentional drift-diffusion model extends to simple purchasing decisions. *Frontiers in psychology*, *3*, 193.

Luce, R. D. (1959). *Individual choice behavior: A theoretical analysis*. Wiley.

McMillen, T., & Holmes, P. (2005).The dynamics of choice among multiple alternatives. *Journal of Mathematical Psychology*, *50*(1), 30–57.

Ratcliff, R. (1978).A theory of memory retrieval. *Psychological Review*, *85*(2), 59–108.

Ratcliff, R., & McKoon, G. (2008).The diffusion decision model: Theory and data for two-choice decision tasks. *Neural computation*, *20*(4), 873–922.

Reutskaja, E., Nagel, R., Camerer, C. F., & Rangel, A. (2011).Search dynamics in consumer choice under time pressure: An eye-tracking study. *American Economic Review*, *101*(2), 900–926.

Shannon, C. E. (1948).A Mathematical Theory of Communication. *The Bell System Technical Journal*, *27*(3), 379–423, 623–656.

Tajima, S., Drugowitsch, J., Patel, N., & Pouget, A. (2019).Optimal policy for multi-alternative decisions. *Nature Neuroscience*, *22*, 1503–1511.

Tsallis, C. (1988). Possible generalization of boltzmann-gibbs statistics. *Journal of Statistical Physics*, *52*, 479–487.