Measuring Consumer Freedom

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Research Question

Household	Product	Price	Quantity
Ann	12345678	7 $	3
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...	...	...	...

How should we make welfare judgments from this data?

Problems of utilitarian approach

- Rationality assumptions
- Model dependence
- Interpersonal comparability
- Missing budget data
- Normative restriction

This paper

- Axiomatizes a measure of freedom by Suppes (1996)
- Applies this measure to consumption data

Literature

- Impartial observer theorem: Harsanyi (1953, 1955), Rawls (1971)
- Entropy axiomatizations: Csiszár (2008)
- Freedom of choice and diversity measures: Suppes (1996), Nehring and Puppe (2009)

The Policy Maker's Decision Problem

- Policy maker decides about what choice procedure to implement for a descendant.
- Policy maker has beliefs about what the descendant will choose.
- Policy maker does not have beliefs about utilities.
- Situation resembles situation behind "realistic" veil of ignorance.

Notation

- Sets in uppercase script, $\mathcal{S}$
- Elements in lowercase $a, b, c, \ldots$
- Outcomes x, y, z
- Real numbers α, β, γ.
- Real numbers λ, μ on the interval $[0, 1]$.
- $0 \ln 0 = 0$.

Choice Procedures

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Terminal Outcomes

We assume there is a set $\mathcal{S}_0 \subset \mathcal{S}$ of terminal outcomes. The set of all procedures $\mathcal{S}$ is finitely generated from $\mathcal{S}_0$ and $\oplus$.
Define the support of a as the set of outcomes $x \in S_0$ chosen with positive probability,

$$supp(a) = \{x \in \mathcal{S}_0 | \exists b, \mu > 0 : a = \mu x \oplus (1 - \mu) b\}.$$

Main Idea

We now impose conditions on how the ancestor should rank the set of procedures, $\mathcal{S}$. Mainly, we allow for the following deviation from von Neumann-Morgenstern:

$$\begin{aligned} & x \sim y \\ \not\Rightarrow \quad & \mu x \oplus (1-\mu) y \sim \mu y \oplus (1-\mu) y \end{aligned}$$

Axiom 1: Rationality

$\succsim$ is a complete, transitive relation on $\mathcal{S}$.

Axiom 2: Continuity

For any $a, b, c \in \mathcal{S}$, the sets $\{\mu | \mu a \oplus (1-\mu) b \succsim c\}$ and $\{\mu | c \succsim \mu a \oplus (1-\mu) b\}$ are closed.

Axiom 3: Disjoint Independence

If $a, b, c \in \mathcal{S}$, $\mu \in (0,1)$, and $(supp(a) \cup supp(b)) \cap supp(c) = \emptyset$ then

$$\begin{aligned} & a \succsim b \\ \Leftrightarrow \quad & \mu a \oplus (1-\mu) c \succsim \mu b \oplus (1-\mu) c \end{aligned}$$

Representation Theorem

Theorem 1

The relation $\succsim$ on $\mathcal{S}$ with at least 8 essential outcomes in $|\mathcal{S}_0|$ fulfills Axioms 1-3 if and only if there exists a continuous, real valued representation $U : \mathcal{S} \to \mathbb{R}$ such that for disjoint a, b,

$$U(\mu a \oplus (1-\mu) b) = \mu^{\alpha} U(a) + (1-\mu)^{\alpha} U(b) + \beta \cdot H_{\alpha}(\mu)$$

$$H_{\alpha}(\mu) = \begin{cases} -\mu \ln \mu - (1-\mu) \ln(1-\mu), & \alpha = 1 \\ -\mu^{\alpha} - (1-\mu)^{\alpha} + 1, & \alpha \neq 1. \end{cases}$$

The representation behaves naturally under compounding, e.g., for $\alpha = \beta = 1$,

$$\begin{aligned}
&U\left(\mu a \oplus (1-\mu)\left(\frac{\lambda}{1-\mu} b \oplus \frac{1-\mu-\lambda}{1-\mu} c\right)\right)\\
=&\mu U(a) - \mu \ln \mu\\
&+ \lambda U(b) - \lambda \ln \lambda\\
&+ (1-\mu-\lambda) U(c) - (1-\mu-\lambda)\ln(1-\mu-\lambda).
\end{aligned}$$

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- Define $H(\lambda) = h(\lambda) + h(1-\lambda)$, then H obeys the fundamental equation of the theory of information, $H(\mu) + g(1-\mu)H(\frac{\lambda}{1-\mu}) = H(\lambda) + g(1-\lambda)H(\frac{\mu}{1-\lambda})$.

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- Caveat: account for support of elements, e.g., $\hat{f}_{supp(a)}(U(a),\mu)$.

Undoing Harsanyi's Special Exemption

Axiom 4: Linearity

Suppose a, b, c and a', b', c' are disjoint. Then if $a \sim b$ and $a' \sim b'$,

$$\begin{aligned} & \mu a \oplus (1-\mu)\left(\frac{\lambda}{1-\mu} b \oplus \frac{1-\lambda-\mu}{1-\mu} c\right) \\ \sim \quad & \mu a' \oplus (1-\mu)\left(\frac{\lambda}{1-\mu} b' \oplus \frac{1-\lambda-\mu}{1-\mu} c'\right) \\ \Leftrightarrow \quad & (\lambda+\mu) a \oplus (1-\lambda-\mu) c \sim (\lambda+\mu) a' \oplus (1-\lambda-\mu) c' \end{aligned}$$

If Axiom 4 holds, then $\alpha = 1$ in the previous representation.

No-choice Indifference

Axiom 5: No-choice Indifference

$x \sim y$ for all $x, y \in \mathcal{S}_0$.

Heavily debated since its introduction by Pattanaik and Xu (1990).
If Axiom 5 holds, then $U(x) = 0$ for all $x \in \mathcal{S}_0$ w.l.o.g..

Unit of Choice

- Choose-and-take: decision to buy product
- Choose-and-spend: decision to allocate dollar

Advantages of choose-and-spend:

- Egg carton problem
- Comparability via money metric

Frequentist Approach

- Consumer dataset $\rightarrow$ choice probabilities
- Frequency of dollar spent $\rightarrow$ probability of spending a dollar

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Dataset

- Nielsen Consumer Panel (2004-2017)
- 40 000 to 60 000 households
- 1.5 M Unique Product Codes (UPCs)
- Retail purchases only - no housing, no services
- Projected to U.S. demographics
- Also, no vegetables and fruits (magnet data)

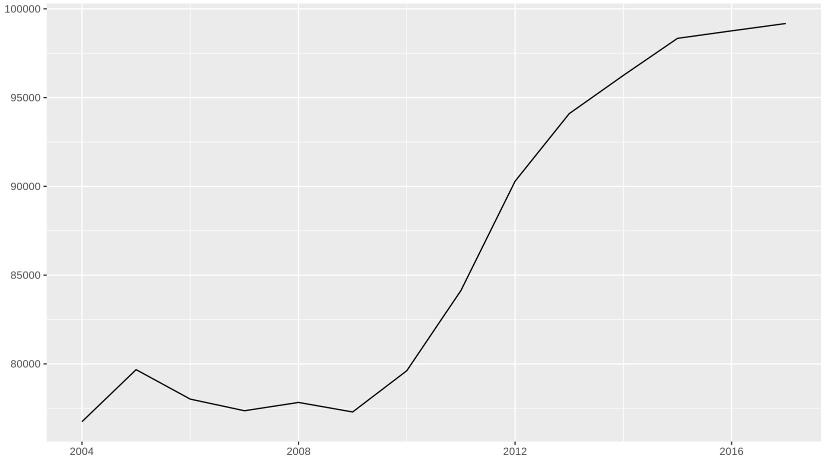


Figure: (Exponential of) Choose-and-spend Entropy by Year

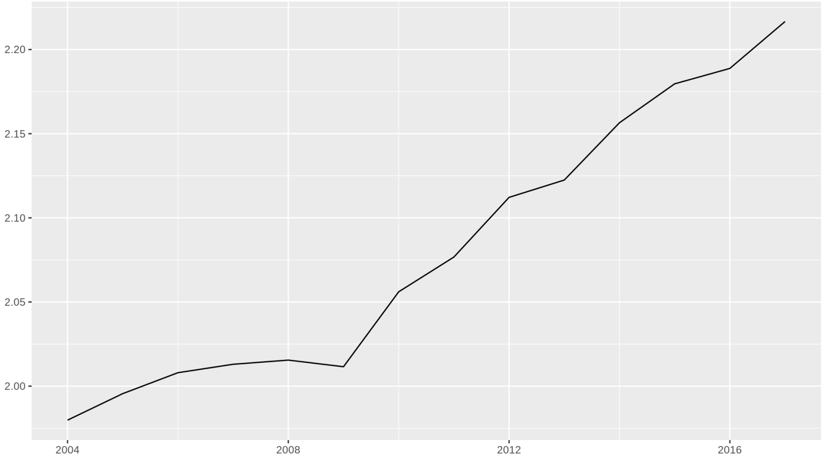


Figure: Product Description Entropy by Year

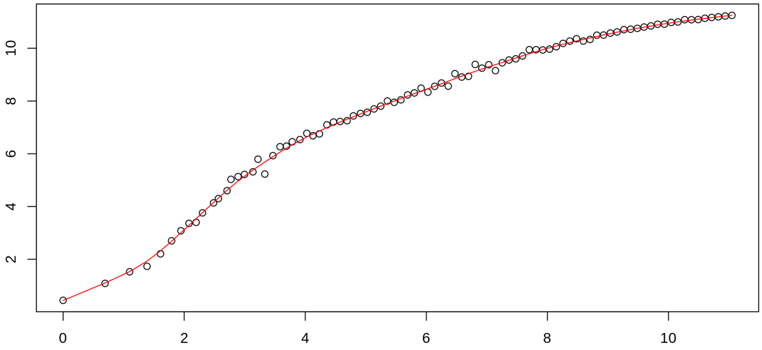


Figure: UPC Entropy 2017 by Logarithm of Sample Size

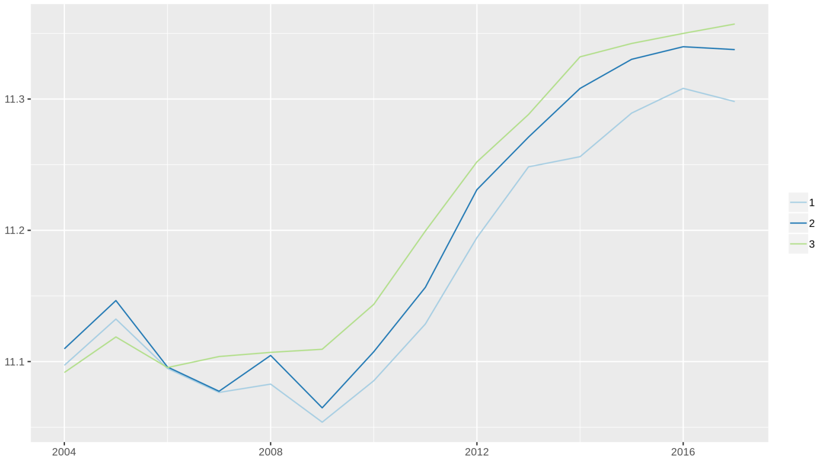
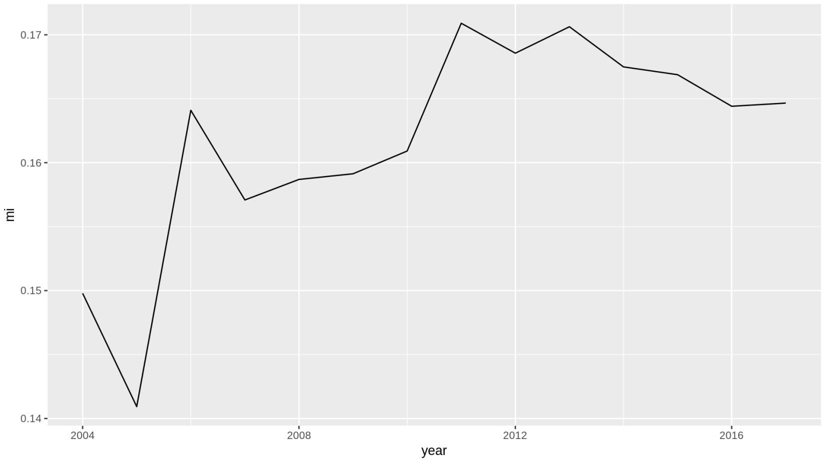


Figure: Choose-and-Spend Entropy by Year and Income Groups

Income as Source of Unfreedom



What else?

- Increase is demand- and supply driven
  - Distribution shape changes
  - Number of UPCs changes
- Online shopping (seems good for consumer freedom)
- Women's education (seems good for consumer freedom)
- Being Asian (seems bad for consumer freedom)
- Being of "other" race (seems good for consumer freedom)

Summary

- We motivated a welfare measure by a representation theorem in the spirit of Harsanyi (1953).
- The axioms provide a cardinal measure of well-being.
- The measure is applicable to consumer data using a choose-and-spend framework.
- Many details remain to be explored!

Disclaimer

Researcher(s) own analyses calculated (or derived) based in part on data from The Nielsen Company (US), LLC and marketing databases provided through the Nielsen Datasets at the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business. The conclusions drawn from the Nielsen data are those of the researcher(s) and do not reflect the views of Nielsen. Nielsen is not responsible for, had no role in, and was not involved in analyzing and preparing the results reported herein.

Bibliography

Csiszár, I. (2008). Axiomatic Characterizations of Information Measures. *Entropy*, *10*, 261–273.

Harsanyi, J. C. (1953). Cardinal Utility in Welfare Economics and in the Theory of Risk-taking. *Journal of Political Economy*, *61*(5), 434–435. doi:10.1086/257416

Harsanyi, J. C. (1955). Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility. *Journal of Political Economy*, *61*, 434–435.

Nehring, K. & Puppe, C. (2009). Diversity. In *Handbook of Rational and Social Choice*. Oxford, UK: Oxford University Press.

Pattanaik, P. K. & Xu, Y. (1990). On ranking opportunity sets in terms of freedom of choice. *Recherches Economiques de Louvain*, *56*, 383–390.

Rawls, J. (1971). *A theory of justice*. Harvard University Press.

Suppes, P. (1996). The nature and measurement of freedom. *Social*

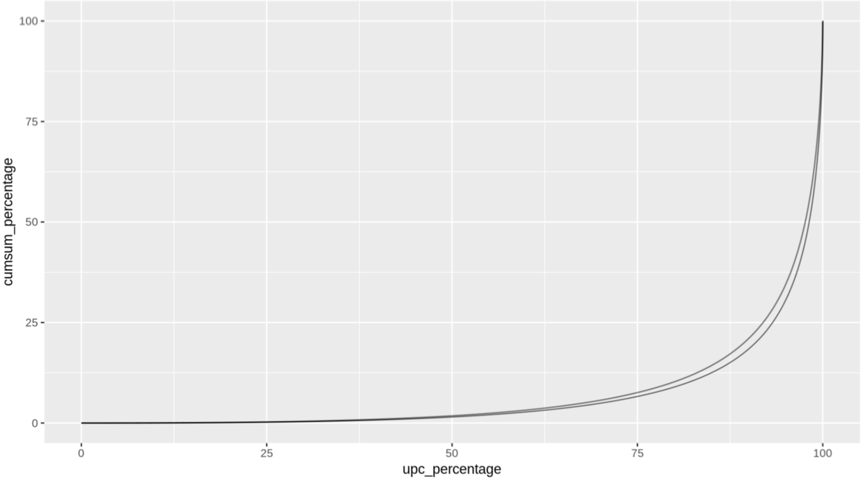


Figure: Cumulative Expenditure Distribution Shift 2004-2017

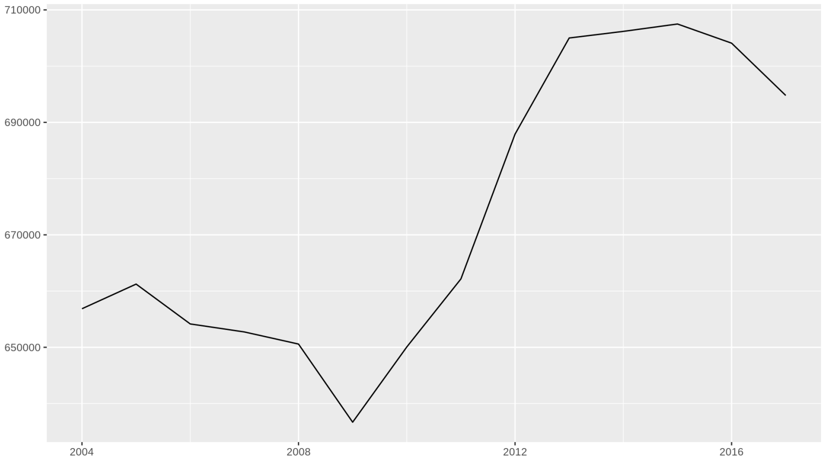


Figure: Number of UPCs Purchased by Year