Measuring Freedom in Games

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Contribution

Impartial observer theorem...

- ...based on observed/expected behavior
- ...without assuming expected utility maximization of individuals
- ...accounting for procedural value

...yielding a measure of freedom of choice...

- ...for interactive settings
- ...that captures the standard intuitions of the literature
- ...while accounting for the control of individuals.

Core Idea

A 50:50 lottery between assigning outcomes “apple” or “banana” is not necessarily indifferent to a 50:50 chance of a player choosing “apple” or “banana”.

Primitives

The policy maker ranks combinations of game forms with information about behavior in the game form. These are called processes (G, θ).

- Game form: $G = (\mathcal{N}, \mathcal{A}, o)$ with $\mathcal{A} = \prod_i \mathcal{A}_i$
- Outcomes $\mathcal{O} = \prod_i \mathcal{O}_i$
- o maps action profiles $\mathcal{A}$ into outcome lotteries $\Delta\mathcal{O}$
- Mixed strategies of i: $\Delta\mathcal{A}_i$
- Information: $\theta \in \prod_{i \in \mathcal{N}} \Delta\Delta\mathcal{A}_i$

Example

	lawful a_2^1	police a_2^2	reject a_2^3
bus, yield, walk a_1^1	u	u	x
bus, yield, cancel a_1^2	u	u	y
bus, refuse, walk a_1^3	v	w	x
bus, refuse, cancel a_1^4	v	w	y
walk a_1^5	z	z	z

Table: Montgomery Bus Game Form

Technical Axioms

- Rationality (completeness and transitivity)
- Continuity (in information about behavior)

Outcome Equivalence

- Sometimes different game forms yield identical behavior.
- If a change in the game form does not change behavior, the policy maker is indifferent to the change.

Example

	lawful a_2^1	police a_2^2	reject a_2^3
bus, yield, walk a_1^1	u	u	x
bus, yield, cancel a_1^2	u	u	y
bus, refuse, walk a_1^3	v	w	x
bus, refuse, cancel a_1^4	v	w	y
walk a_1^5	z	z	z

Table: Montgomery Bus Game Form

Lottery Independence

- Some processes give individuals no choice.
- These are effectively lotteries.
- The policy maker should obey the vNM independence axiom for these.

Strategy Independence

- Two processes may agree on the game form and on information about the choices on a subset of strategies of a subset of players.
- Letting nature instead choose between these strategies does not change the comparative ranking of the processes.

Example

- Suppose we compare two processes that completely agree on the information about player 2's strategies.

	lawful a_2^1	police a_2^2	reject a_2^3
bus, yield, walk a_1^1	u	u	x
bus, yield, cancel a_1^2	u	u	y
bus, refuse, walk a_1^3	v	w	x
bus, refuse, cancel a_1^4	v	w	y
walk a_1^5	z	z	z

Table: Montgomery Bus Game Form

Example

- Suppose we compare two processes that completely agree on the information about player 2's strategies.
- Both the behavior and the conditional probabilities of the outcomes achieved by player 2 are identical.

	lawful a_2^1	police a_2^2	reject a_2^3
bus, yield, walk a_1^1	u	u	x
bus, yield, cancel a_1^2	u	u	y
bus, refuse, walk a_1^3	v	w	x
bus, refuse, cancel a_1^4	v	w	y
walk a_1^5	z	z	z

Table: Montgomery Bus Game Form

Example

- Suppose we compare two processes that completely agree on the information about player 2's strategies.
- Both the behavior and the conditional probabilities of the outcomes achieved by player 2 are identical.
- Then letting nature determine player 2's choice does not change the preference between the processes.

	lawful a_2^1	police a_2^2	reject a_2^3
bus, yield, walk a_1^1	u	u	x
bus, yield, cancel a_1^2	u	u	y
bus, refuse, walk a_1^3	v	w	x
bus, refuse, cancel a_1^4	v	w	y
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Table: Montgomery Bus Game Form

Subprocess Monotonicity

- Thanks to the work of Mailath, Samuelson, and Swinkels (1993), there exists a definition of subgames in strategic game forms.
- Via Bayesian updating of θ, we can define subprocesses.
- Improving a subprocess with unrelated outcomes from the remainder of the game should improve the overall process.

Example

Suppose the outcomes "on the bus" are disjoint from the outcomes "off the bus". Subprocess monotonicity requires that if we improve the subprocess "on the bus", the overall process improves.

	lawful a_2^1	police a_2^2	reject a_2^3
bus, yield, walk a_1^1	u	u	x
bus, yield, cancel a_1^2	u	u	y
bus, refuse, walk a_1^3	v	w	x
bus, refuse, cancel a_1^4	v	w	y
walk a_1^5	z	z	z

Table: Montgomery Bus Game Form

Representation Theorem

Theorem 1

Suppose for every player there are at least four essential pairs of outcomes. The policy maker's preferences on the set of processes fulfill Axioms 1-6 if and only if there exists a continuous representation such that

- *the policy criterion is additively separable across individuals, and*
- *for every individual the policy maker sums an expected valuation of the outcomes and the expected control (in terms of mutual information) of the strategies over the outcomes.*

Representation Theorem

Theorem 1

Suppose for every player i there are at least four essential pairs of outcomes. The relation $\succsim$ *on the set of processes* $\mathcal{P}$ *fulfills Axioms 1-6 if and only if there exists a continuous, real valued representation* $U : \mathcal{P} \to \mathbb{R}$ *such that*

$$U[G, \theta] = \sum_{i \in \mathcal{N}} U_i[G, \theta] \tag{1}$$

$$U_i[G, \theta] = \sum_{\mu_i} \theta_i[\mu_i] \sum_{x_i \in \mathcal{O}_i} \rho_{G,\theta}|\mu_i[x_i] \left(v_i[x_i] + d_i \ln \left[\frac{\rho_{G,\theta}|\mu_i[x_i]}{\rho_{G,\theta}[x_i]} \right] \right) \tag{2}$$

$\rho_{G,\theta}$ $(\rho_{G,\theta}|\mu_i)$ is the (conditional) probability measure of outcomes (given strategy μ_i).

Applications

The measure/criterion should be able to inform us also in practice. In several applications this is exemplified.

- Income Taxation (in paper): Tax progression is bad for consumer freedom but can be beneficial to increase labor freedom (control to determine hours worked).
- Consumer Freedom (with Tzu-Ying Chen): Using retail scanner data, we estimate an increase of freedom of choice in the U.S. from the equivalent of about 80 000 equally likely chosen products to about 100 000 products between 2010 and 2017 but no change between 2004 and 2009.
- Monetary Policy (with Hao Wu): In a standard new Keynesian model, a central bank maximizing consumer freedom emphasizes price stability over output stability in the optimum.

Summary

Freedom of choice arises naturally as a normative criterion when utility information is unavailable. It can be quantified based on observable data. Information-theoretic measures are a good starting point for estimation and arise naturally from separability conditions in strategies and subgames.