Preference for Flexibility from Incomplete Resolution of Uncertainty

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Introduction

Introduction

Preference for flexibility

- refers to a strict preference for a menu over its subsets.
- is commonly modelled with subjective states.

What we do:

- Explain preference for flexibility by hidden information.
- Characterize the representation for the above model.
- Develop a method to elicit hidden information sets.

Timeline

Have an information partition unknown to the analyst.

$\downarrow$

Stage 1: Choose among information acts.

$\downarrow$

Learn from the hidden information.

$\downarrow$

Stage 2: Choose from the corresponding menu.

$\downarrow$

True state realizes.

Notation

Notation

$\mathcal{X}$	outcome space
$\mathcal{S}$	state space
$\mathcal{E}$	σ-algebra of events on $\mathcal{S}$
$b_E : E \to \mathcal{X}$	conditional subsequent act, finite image
m_E	conditional menu, finite collection of b_E
$\mathcal{M}_E$	the set of all m_E
$\mathcal{M}$	$\bigcup_{E \in \mathcal{E}} \mathcal{M}_E$

Notation: Information Act

An information act is a mapping $a : \mathcal{J} \to \mathcal{M}$, where

- $\mathcal{J}$ is a finite information partition of $\mathcal{S}$;
- $a(E) \in \mathcal{M}_E$ for all $E \in \mathcal{J}$.

Useful notation:

- $f = m^1_{E^1} \cdots m^{p-1}_{E^{p-1}} m^p_{E^p}$ and $g = n^1_{F^1} \cdots n^{q-1}_{F^{q-1}} n^q_{F^q}$.
- $\iota(f) = \{E^1, \ldots, E^p\}$ is the information partition.
- $f_G g = m^1_{E^1 \cap G} \cdots m^p_{E^p \cap G} n^1_{F^1 \cap G^c} \cdots n^q_{F^q \cap G^c}$.

An event E is nonnull if $\{\alpha\}_E\{\beta\} \succ \{\beta\}_E\{\beta\}$ for some $\alpha, \beta \in \mathcal{X}$.

Example

Let $m = \{b^1, b^2\}$, where

$$b^1(s) = \begin{cases} 2 & \text{if } s \in F \\ 0 & \text{if } s \in F^c \end{cases} \quad \text{and} \quad b^2(s) = \begin{cases} 0 & \text{if } s \in E \\ 1 & \text{if } s \in E^c \end{cases}.$$

No hidden information	Hidden information $\{E, E^c\}$
$\{b^1\}_{\mathcal{S}} \succ \{b^2\}_{\mathcal{S}}$	$\{b^1\}_{\mathcal{S}} \succ \{b^2\}_{\mathcal{S}}$
$m_{\mathcal{S}} \sim \{b^1\}_{\mathcal{S}}$	$m_{\mathcal{S}} \succ \{b^1\}_{\mathcal{S}}$
$m_E m \succ m_{\mathcal{S}}$	$m_E m \sim m_{\mathcal{S}}$
$\{b^1\}_E \{b^2\} \sim m_E m$	$\{b^1\}_E \{b^2\} \sim m_E m$

Axioms

Axioms: Savage's Six

Weak Order

- $\succsim$ is complete and transitive.

Sure-Thing Principle

- $h_E f \succsim h_E g \iff k_E f \succsim k_E g$.
- Related to additivity.
- Define $\succsim_E$: $f \succsim_E g \iff f_E h \succsim_E g_E h$.

Monotonicity

- $\{\alpha\} \succsim \{\beta\} \iff \{\alpha\}_E f \succsim \{\beta\}_E f$ for nonnull E.
- Final outcomes are valued independently.

Axioms: Savage's Six

Likelihood Outcome Independence

- $\{\alpha\} \succ \{\beta\}$, $\{\gamma\} \succ \{\delta\}$ implies
$$\{\alpha\}_E\{\beta\} \succsim \{\alpha\}_F\{\beta\} \Leftrightarrow \{\gamma\}_E\{\delta\} \succsim \{\gamma\}_F\{\delta\}.$$
- Guarantee the existence of a likelihood relation $\succsim^*$:
$$E \succsim^* F \Longleftrightarrow \{\alpha\}_E\{\beta\} \succsim \{\alpha\}_F\{\beta\} \text{ for some } \{\alpha\} \succ \{\beta\}.$$

Nontriviality

- $\exists\, \alpha, \beta \in \mathcal{X}$ such that $\{\alpha\} \succ \{\beta\}$.

Continuity

- $f^k \rightsquigarrow f$, $g^k \rightsquigarrow g$, and $f^k \succsim g^k\ \forall\, k$ implies $f \succsim g$.

Axioms: Knowledge

Instrumental Knowledge Property: $\{b\}_{E \cup F} f \sim \{b\}_E \{b\}_F f$.

Consistency of Hidden Knowledge

- Let $\{\alpha\} \succ \{\beta\}$. If an event I and $n = \{b^1, b^2\}$ where

$$b^1(s) = \begin{cases} \alpha, & \text{if } s \in I \\ \beta, & \text{if } s \in I^c \end{cases} \quad \text{and} \quad b^2(s) = \begin{cases} \beta, & \text{if } s \in I \\ \alpha, & \text{if } s \in I^c \end{cases}$$

  satisfy $n \sim n_I n$, then for all m and E, $m \sim_E m_I m$.
- Such I is called a hidden identified set.

Axioms: Hidden Indirect Utility Property

Indirect Utility Property: $\{b\} \succsim_E m \implies \{b\} \cup m \sim_E \{b\}$.

Hidden Indirect Utility Property

- $m \cup n \succsim_E m$ and $m \cup n \succsim_E n$.
- If $m \cup n \succ_E m$ and $m \cup n \succ_E n$, then there exists an hidden identified set I such that $m \cup n \sim_{E \cap I} m$ and $m \cup n \sim_{E \cap I^c} n$.
- Explain preference for flexibility w/o subjective state space.

Representation

Representation

Theorem

There exist

- $U : \mathcal{A} \to \mathbb{R}$,
- $u : \mathcal{X} \to \mathbb{R}$ unique up to affine transformations,
- a unique probability measure $\mu : \mathcal{E} \to \mathbb{R}$, and
- a hidden information sigma algebra $\mathcal{H}$

such that $U(a) \geq U(a')$ if and only if $a \succsim a'$ and

$$U(a) = \sum_{E \in \iota(a)} \mu(E) \int_E \max_{b \in a(E)} v_b \, \mathrm{d}\mu^{\mathcal{H}|E},$$

where $v_b(H) = \int_H u \circ b \, \mathrm{d}\mu^{\mathcal{E}|H}$.

Representation: Sketch of Proof

Step 1. Show an additive representation exists.

Step 2. Elicit the unique probability measure.

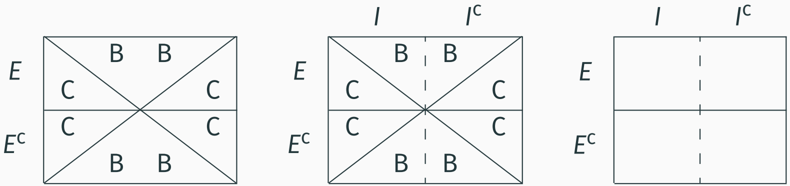
Step 3. Show the set of all hidden identified sets is a σ-algebra.

Step 4. Apply Randon-Nikodym theorem

Step 5. Use hidden indirect utility property

Elicitation of Hidden Information

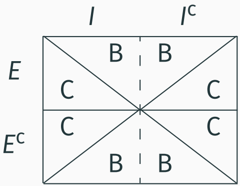
Elicitation of Hidden Information: Setting



- E and E^c : information offered by the analyst.
- I and I^c : the agent's hidden information.
- B : the region where $\{b(s)\} \succ \{c(s)\}$.
- C : the region where $\{c(s)\} \succ \{b(s)\}$.
- Suppose $\{b, c\}_E\{b\} \succ \{b\}_E\{b\}$ and $\{b, c\}_E\{b\} \succ \{c\}_E\{b\}$.
  $\Rightarrow$ assume the agent chooses b in $E \cap I$ and c in $E \cap I^c$.

Elicitation of Hidden Information: Method

	I	I^c
E	b	c
E^c	b	b

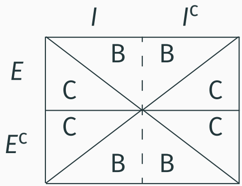


Attempt to identify $B \cap E \cap I^c$:

- Pick $\varepsilon \in B \cap E$ and offer $\{b, c\}_{E \setminus \varepsilon}\{b\}$.
- Compare the marginal change of value.

Elicitation of Hidden Information: Case 1

	I	I^c
E	b	c
E^c	b	b

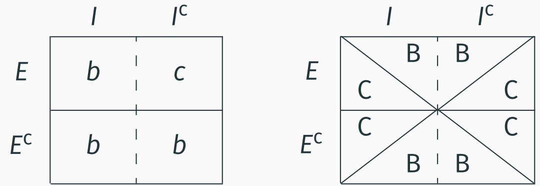


Case 1: $\varepsilon \in B \cap E \cap I^c$.

- Decrease the value of b in $E \cap I^c$.
- The agent still chooses c in $E \cap I^c$.

$$\begin{aligned} v\big(\{b,c\}, E \setminus \varepsilon\big) &= v\big(\{b\}, E \cap I\big) + v\big(\{c\}, E \cap I^c \setminus \varepsilon\big) \\ &= v\big(\{b\}, E \cap I\big) + v\big(\{c\}, E \cap I^c\big) - v\big(\{c\}, \varepsilon\big) \\ &= v\big(\{b,c\}, E\big) - v\big(\{c\}, \varepsilon\big). \end{aligned}$$

Elicitation of Hidden Information: Case 2



Case 2: $\varepsilon \in B \cap E \cap I$.

- Decrease the value of b in $E \cap I$.
- May cause preference reversal in $E \cap I$.

$$v\big(\{b, c\}, E \setminus \varepsilon\big) = v\big(\{b, c\}, E \cap I \setminus \varepsilon\big) + v\big(\{c\}, E \cap I^c\big)$$

$$= \begin{cases} v\big(\{b, c\}, E\big) - v\big(\{b\}, \varepsilon\big), & \text{if } b \text{ is chosen in } E \cap I \setminus \varepsilon, \\ v\big(\{c\}, E\big) - v\big(\{c\}, \varepsilon\big), & \text{if } c \text{ is chosen in } E \cap I \setminus \varepsilon. \end{cases}$$

Elicitation of Hidden Information: Conclusion

It remains to compare

$$v\big(\{b,c\},E\big) - v\big(\{c\},\varepsilon\big) \quad \text{and} \quad \begin{cases} v\big(\{b,c\},E\big) - v\big(\{b\},\varepsilon\big) \\ v\big(\{c\},E\big) - v\big(\{c\},\varepsilon\big) \end{cases}.$$

Observe that

- $\varepsilon \subseteq B \cap E \Longrightarrow v\big(\{b\},\varepsilon\big) > v\big(\{c\},\varepsilon\big)$.
- $\{b,c\}_E\{b\} \succ \{c\}_E\{b\} \Longrightarrow v\big(\{b,c\},E\big) > v\big(\{c\},E\big)$.

$C \cap E \cap I$ can be identified in a similar way.

Discussion

An interesting extension is to consider hidden actions.

- Say, $\forall E, \{b\} \cup m \sim_E m$, but $\{b\}$ dominates m somewhere.

How to test the axioms?

What if knowledge has its own intrinsic value?

Questions?