# Preference for Verifiability

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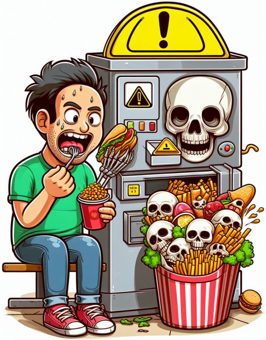
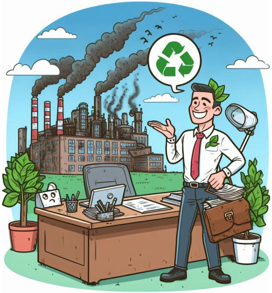



# Contribution

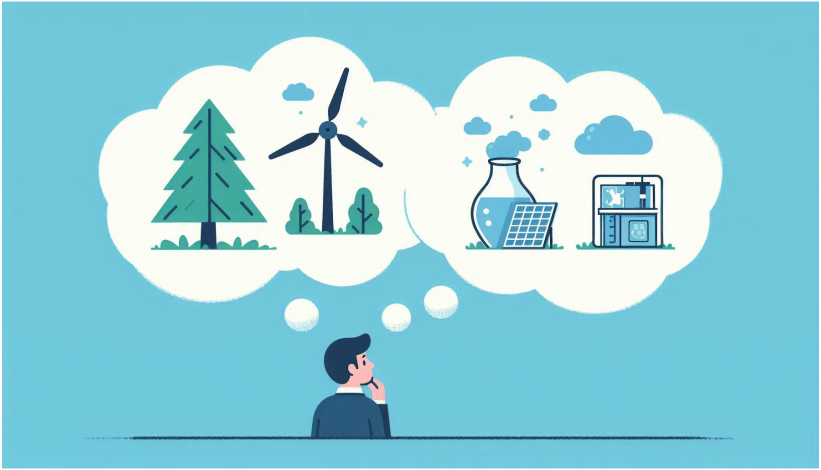
- Parsimonious theory of greenwashing.
  - Identification of a principal-agent relationship from behavior
  - We only observe the agent's preferences over acts
- Decision theory with unobservable outcomes.
  - Unobservable outcomes provide motivation for relaxing STP to Comonotonicity/Certainty independence
  - In this paper: DM cares about what is ex-post certain

# General Idea

- Decision theory implicitly assumes that outcomes are observable.
- Decision makers may care about unobservable outcomes (e.g., due to altruism, legitimacy, etc.).
- Decision makers may care about verifying/obfuscating whether good/bad outcomes have been achieved.
- Analysts may care about identifying such decision models (e.g., no greenwashing)

# Relation to Literature

- Dual-self ambiguity aversion: Chandrasekher et al. (2022)
- Cominimum Additivity: Kajii et al. (2007), Kajii et al. (2009)
- Contract theory: Bull and Watson (2004)
- Definitions of greenwashing: de Freitas Netto et al. (2020)
- Formal models of greenwashing: Wu et al. (2020)
- Green products: Groening et al. (2018)

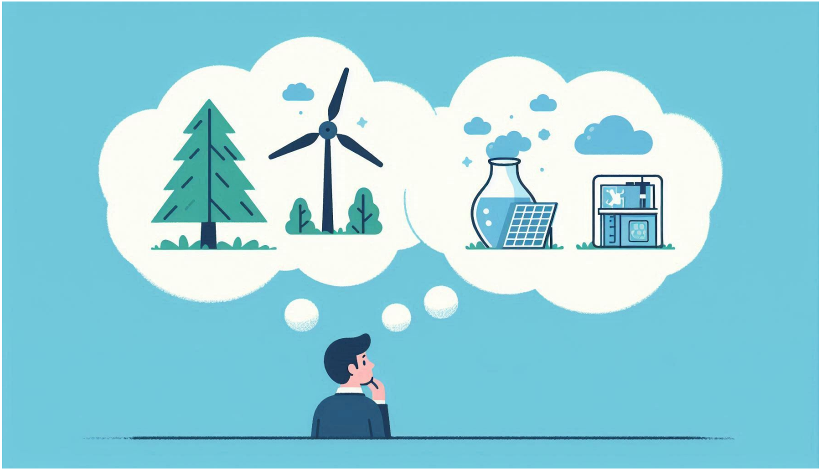


# Carbon Reduction Decision Problem

- Carbon emissions (outcomes) not directly observable
- Efficacy of offset/reduction methods uncertain (depends on state of the world)
- Firm chooses between different carbon offset/reduction methods (acts)
- Information about state of the world released afterwards (verifiable events)

# Example: States, Acts, Outcomes

- States $\mathcal{S} = \{s, t, u\}$
- Outcomes $\mathcal{X} = \mathbb{R}$: CO2 emission reduction
- Acts $\mathcal{A}$: emission reduction methods

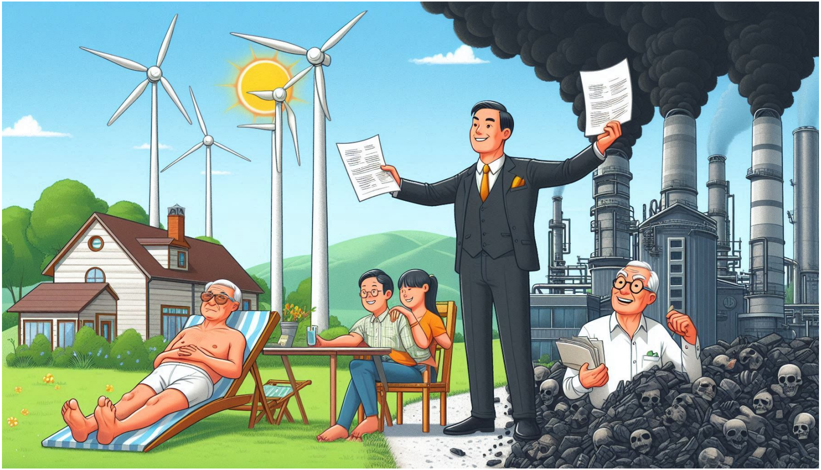


# Available Alternatives

Assume total expenditure on $CO_2$ mitigation is fixed. Firm chooses one of three alternatives:

- Nature based carbon removal: ex-ante uncertain but ex-post verifiable
- RECs: ex-ante and ex-post uncertain reduction
- Emission reduction: low but ex-ante certain reduction.

# Substitution effect in RECs?



# Example: States of the World

Three states of the world:

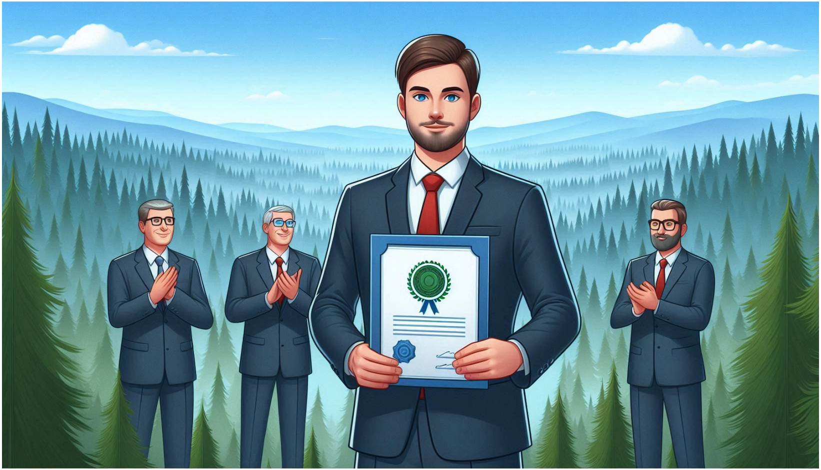
- $s$: high availability of offsets, substitution
- $t$: high availability of offsets, no substitution
- $u$: low availability of offsets

Assume $\{s, t\}$ and $\{s, t, u\}$ are ex post verifiable.

# Example: Decision Matrix

| | $s$ | $t$ | $u$ |
|---|---|---|---|
| Trees | 70 | 70 | 10 |
| RECs | 60 | 100 | 10 |
| Efficiency | 40 | 40 | 40 |

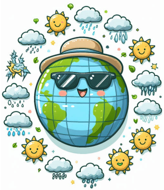
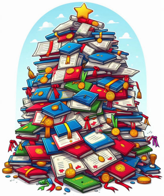
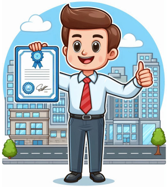
# Preference for Verification



# Preference for Verification

- Firm wants to prove to stakeholders that they *have definitely* offset a certain amount of carbon emissions
- Firm fears to have no proof of offset.
- Example: Apple chose nature based carbon removal in UN Race to Zero Campaign

# Preference for Verification Timing


# Expected Verification Utility

## Definition (Expected Verification Utility)

A preference relation $\succsim$ on $\mathcal{A}$ is an *expected verification utility* if there exists a nonempty set of events $\mathcal{V} \subseteq \mathcal{E}$, closed under intersection, a probability measure $\mu : \mathcal{E} \to [0, 1]$, and a convex-valued utility function $u : \mathcal{X} \to \mathbb{R}$ such that

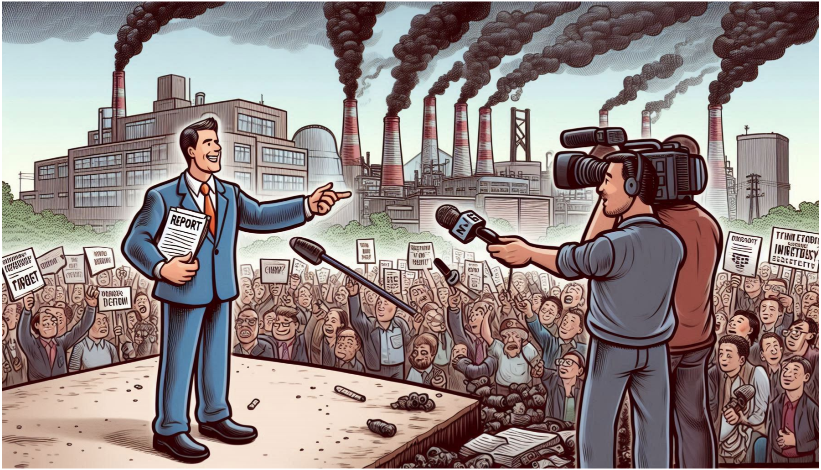
$$U(a) = \int_{s \in \mathcal{S}^*} \max_{E \in \mathcal{V}: s \in E} \min_{t \in E} u(a(t)) d\mu \tag{1}$$

represents $\succsim$.

# Interpretation

- If state $s \in E \in \mathcal{V}$ obtains, then DM receives a proof that $E$ obtains.
- DM can use the proof to show stakeholders that at least utility $\min_{s \in E} u(a(s))$ has been achieved.
- DM can combine multiple proofs.

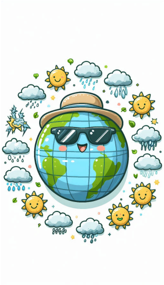
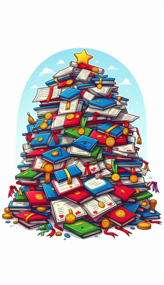
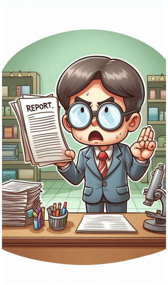
# Preference for Obfuscation



# Preference for Obfuscation

- Firm wants to point out to stakeholders that they *might have* offset a certain amount of carbon emissions
- Firm fears someone has proof how much they actually offset.
- Example: Foxconn chose RECs in UN Race to Zero Campaign

# Preference for Obfuscation Timing

# Expected Obfuscation Utility

## Definition (Expected Obfuscation Utility)

A preference relation $\succsim$ on $\mathcal{A}$ is an *expected obfuscation utility* if there exists a nonempty set of events $\mathcal{V} \subseteq \mathcal{E}$, closed under intersection, a probability measure $\mu : \mathcal{E} \to [0, 1]$, and a convex-valued utility function $u : \mathcal{X} \to \mathbb{R}$ such that

$$U(a) = \int_{s \in \mathcal{S}^*} \min_{E \in \mathcal{V}: s \in E} \max_{t \in E} u(a(t)) d\mu \qquad (2)$$

represents $\succsim$.

# Interpretation

- If state $s \in E \in \mathcal{V}$ obtains, then the stakeholder receives a proof that $E$ obtains.
- Stakeholder can use the proof to show that at most utility $\max_{s \in E} u(a(s))$ has been achieved.
- Stakeholder can combine multiple proofs.

# Verifiable Events

- $\mathcal{V}$ is a $\pi$-system
- If I can prove that $E$ is true and I can prove that $F$ is true then I can prove that $E \cap F$ is true.
- If all I care about is the worst possible outcome on an event, then I don't ever need to show that $E \cup F$ is true.

# Structural Assumption

**Axiom (Biseparable Preference (Ghirardato & Marinacci, 2001))**

$\succsim$ is a *biseparable preference* if there exists a monotonic representation $U : \mathcal{A} \to \mathbb{R}$, an event $E \in \mathcal{E}^{**}$, a set function $\mu : \mathcal{E} \to [0, 1]$, such that for all $\gamma \succsim \beta$ and all events $F \in \mathcal{E}$:

$$U(\gamma F \beta) = \mu(F)U(\gamma) + (1 - \mu(F))U(\beta) \tag{3}$$

$U(\mathcal{X})$ is convex.

# Preference Averages

**Definition (Preference Average (Ghirardato et al., 2003))**

For all $x, y \in \mathcal{X}$ with $x \succsim y$, $z$ is a preference average of $x$ and $y$ if $xEy \sim [xEz]E[zEy]$. $z$ is denoted by $1/2x \oplus 1/2y$.

Define preference averages of acts pointwise: $c = 1/2a \oplus 1/2b$ if in all states $s$ we have $c(s) = 1/2a(s) \oplus 1/2b(s)$.

# Comonotonicity

**Definition (Comonotonic Acts)**

Acts $a, b \in \mathcal{A}$ are *comonotonic* if for all $s, s' \in \mathcal{S}$,

- $a(s) \succ a(s') \Rightarrow b(s) \succsim b(s')$, and
- $b(s) \succ b(s') \Rightarrow a(s) \succsim a(s')$.

# Comontonic Independence

## Axiom (Comonotonic Independence)

$\succsim$ fulfills *comonotonic independence* if for all comonotonic $a, b, c$, $a \succsim b$ if and only if $1/2a \oplus 1/2c \succsim 1/2b \oplus 1/2c$.

# Supermodularity

## Axiom (Supermodularity)

$\succsim$ fulfills *supermodularity* if for all events $E$, $F$, and all outcomes $\gamma \succ \beta \in \mathcal{X}$,
$1/2[\gamma E \cup F \beta] \oplus 1/2[\gamma E \cap F \beta] \succsim 1/2[\gamma E \beta] \oplus 1/2[\gamma F \beta]$

# Submodularity

## Axiom (Submodularity)

$\succsim$ fulfills *submodularity* if for all events $E, F$, and all outcomes $\gamma \succ \beta \in \mathcal{X}$,
$1/2[\gamma E \cup F \beta] \oplus 1/2[\gamma E \cap F \beta] \precsim 1/2[\gamma E \beta] \oplus 1/2[\gamma F \beta]$

# Critical Events

**Definition (Critical Event)**

An event $E$ is *min-increasing* if $\gamma E \beta \not\sim \gamma E - F \beta$ for all nonnull events $F \subset E$ and some outcomes $\gamma \succ \beta$.

In an expected verification utility, an event $E$ is min-increasing if there exists a subset of $\mathcal{V}$ containing only subsets of $E$ that jointly cover $E$.

# Critical Events

**Definition (Critical Event)**

An event $E$ is *max-increasing* if $\beta_{E\cup F}\gamma \succ \beta_E\gamma$ for all nonnull events $F \subset \overline{E}$ and some outcomes $\gamma \succ \beta$.

In an expected obfuscation utility, an event $E$ is max-increasing if there exists a subset of $\mathcal{V}$ containing only subsets of $E$ that jointly cover $E$.

# Critical Events

- In the two representations, min-increasing events and max-increasing events play the exact same role.
- I therefore simply refer to these as *critical* events.
- Critical means that either min-increasing or max-increasing holds.

# Critical Event Modularity

**Axiom (Critical Event Modularity)**

$\succsim$ fulfills *critical event modularity* if for all critical events $E$, $F$, and any event $A \subseteq E \cup F$,

1. $E \cap F$ is a critical event,
2. $E \cup F$ is a critical event, and
3. $1/2[\gamma A \beta] \oplus 1/2[\gamma A \cap E \cap F \beta] \sim 1/2[\gamma A \cap E \beta] \oplus 1/2[\gamma A \cap F \beta]$

**Theorem (Verification Representation Theorem)**

*Suppose $\succsim$ is a biseparable preference with representation $U$ and set function $\mu$. Then the following statements are equivalent:*

1. *$\succsim$ fulfills Comonotonic Independence, Supermodularity, and Critical Event Modularity.*
2. *$\succsim$ is an expected verification utility.*

**Theorem (Obfuscation Representation Theorem)**

*Suppose $\succsim$ is a biseparable preference with representation $U$ and set function $\mu$. Then the following statements are equivalent:*

1. *$\succsim$ fulfills Comonotonic Independence, Submodularity, and Critical Event Modularity.*
2. *$\succsim$ is an expected obfuscation utility.*

# Uniqueness

## Uniqueness

Suppose $\succsim^1$ and $\succsim^2$ are expected verification utilities. Then $a \succsim^1 b \Leftrightarrow a \succsim^2 b$ for all $a, b \in \mathcal{A}$ if and only if:

- $U^1 = \theta U^2 + \phi$,
- $cl_\cup(\mathcal{V}^1) = cl_\cup(\mathcal{V}^2)$,
- $\forall E \in \mathcal{V}^1 \cap \mathcal{V}^2 : \mu^1(E) = \mu^2(E)$.

# Comparative Statics

## Comparative Statics

Suppose $\succsim^1$ and $\succsim^2$ are expected verification utilities with $\gamma \succ^1 \beta$ and $\gamma \succ^2 \beta$ and identical null events. Then the following statements are equivalent:

1. $\mathcal{V}^1 \subseteq \mathcal{V}^2$.
2. $\gamma E \beta \sim^2 \gamma(E - F)\beta$ implies $\gamma E \beta \sim^1 \gamma(E - F)\beta$

# Other Comparative Statics & Results

- Comparative risk aversion well defined via $\oplus$. Unrelated to critical events.
- Ambiguous events: non-critical events.
- Welfare: deviation from EU maximization depends on decision problem.
- Information preference: expected verification (obfuscation) utility seeks larger (smaller) $\mathcal{V}$.

# Cominimum Independence

## Cominimum Acts

Two acts $a, b$ are $\mathcal{C}$-*cominimum* if for all $E \in \mathcal{C}$,
$\arg_{s \in E} \min_{\succsim} a(s) \cap \arg_{s \in E} \min_{\succsim} b(s) \neq \emptyset$.

## Cominimum Independence

A preference $\succsim$ is $\mathcal{C}$-cominimum independent if it fulfills the independence axiom for $\mathcal{C}$-cominimum independent acts.

# Characterization with exogeneous $\mathcal{V}$

- Theorem 2 of Kajii et al. (2007) characterizes a similar functional form when verifiable events are given/known.
- $\mathcal{V}$-cominimum independence provides a direct behavioral test when the information partition of the state space is objective, i.e., if it is known what environmental studies will be performed.

# A decision theory for unobservable outcomes

- Theoretical treatments of unobservable outcomes (e.g., in the marketing literature) not satisfactory
- Unobservable outcomes are ubiquitous:
  - Donations/charitable giving
  - Green products
  - Toxicity, long-term health effects
  - Products with ex-post uncertainty about effect on well-being
- Much research and applications to be done.

# Concluding Remarks

- First attempt at a decision theory with unobservable outcomes.
- Beliefs and verifiable events (mostly) identifiable.
- Model can be used to identify greenwashing vs. legitimacy-seeking in the context of CO2 mitigation.

Thank You!

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